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A Decomposition Algorithm for Single and Multi-Objective Integrated Market Selection and Production Planning

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We study an integrated market selection and production planning problem. There is a set of markets with deterministic demand and each market has a certain revenue, which is obtained if the market's demand is satisfied throughout a planning horizon. The demand is satisfied with a production scheme that has a lot-sizing structure. The problem is to decide on which markets' demand to satisfy and plan the production simultaneously. We consider both single and multi-objective settings. The single objective problem maximizes the profit, while the multi-objective problem includes the maximization of the revenue and the minimization of the production cost objectives. We develop a decomposition-based exact solution algorithm for the single objective setting, and show how it can be used in a proposed three-phase algorithm for the multi-objective setting. The master problem chooses a subset of markets, and the subproblem calculates an optimal production plan to satisfy the selected markets' demand. We investigate the subproblem from a cooperative game theory perspective to devise cuts and strengthen them based on lifting. We also propose a set of valid inequalities and preprocessing rules to improve the proposed algorithm. We test the efficacy of our solution method over a suite of problem instances and show that our algorithm substantially decreases solution times for all problem instances.

Key words: Market Selection, Lot-Sizing Problem, Decomposition Algorithm, Multi-Objective Optimization, Cooperative Game Theory

History:

1. Introduction

The economic lot-sizing problem is a production planning problem where the goal is to satisfy a set of deterministic dynamic demand over a planning horizon at minimum production cost. In this traditional problem, the set of demands to be satisfied are predetermined and there is no option for partial satisfaction by choosing some percentage of the demand. However, in today's competitive business environment, global companies make decisions not only on the supply side but also on the demand side (Geunes et al. 2011). Specifically, these companies select the set of markets whose demands they would like to satisfy instead of satisfying a predetermined deterministic demand. Hence, the problem has two decision stages: first, to determine the markets whose demand will be satisfied, and then, plan the lot-sizing decisions to satisfy the whole demand. Since the lot-sizing decisions are affected by the total demand to be satisfied, it is important to select the markets wisely to satisfy their demands. Hence, the market selection and production decisions need to be made simultaneously.

Van den Heuvel et al. (2012) consider a production planning problem where a set of markets with known demands and corresponding revenues exists. The problem is to select the markets whose demands will be fully satisfied through a single product that has a lot-sizing production cost structure. They develop an integer programming formulation for the economic lot-sizing problem with market selection, which they refer to as the market selection problem. Then, they prove that the problem is NP-hard, introduce polynomially solvable special cases and a heuristic for the general case. The problem that we consider focuses on a similar problem as Van den Heuvel et al. (2012) within a single and a multi-objective problem setting. More specifically, we consider a lot-sizing problem with market selection where the objectives are (*i*) the maximization of the net profit that is calculated by subtracting the total production cost from the revenue obtained by demand satisfaction (in the single-objective setting), (*ii*) the maximization of the revenue that we obtain by selecting the markets whose demand will be satisfied throughout the planning horizon, and (*iii*) the minimization of the total production and inventory holding cost of items that satisfy the selected markets' demands.

A version of the market selection problem, which is called order selection problem, is first studied by Geunes et al. (2002). They consider a production planning problem where there exists a set of outstanding orders with associated amounts, delivery periods and profits over a planning horizon, and the marketing and production planning departments together select from these orders the ones they wish to satisfy in order to maximize the net profit from the production. If it is not profitable to satisfy a certain amount of demand, then this amount of the order can be rejected. Hence, revenue is obtained only from the portion of the demand that is satisfied. Then, Geunes et al. (2011) consider a similar problem where in the first stage they select the markets whose demands will be fully satisfied, and in the second stage, they minimize the total production cost and the lost revenues that arise from rejecting to serve the markets in the first stage. They propose approximation algorithms for solving this economic lot-sizing problem with market selection.

Market selection in the context of Economic Order Quantity (EOQ) (Geunes et al. 2004, Geunes 2012), and newsvendor problems (Taaffe et al. 2008, Chahar and Taaffe 2009, Strinka et al. 2013) are also studied in the literature. Geunes (2012) provides the model and solution methods for a set of EOQ problems where it is possible to choose whether to satisfy multiple markets. Taaffe et al. (2008) consider the newsvendor problem where the price of the item is market-dependent and the demands of the markets depend on the marketing effort applied. In this setting the firm decides on which markets to serve prior to procuring the items. The proposed model implicitly accounts for inventory pooling across markets, which in turn decreases the safety stock costs.

Market selection decisions are not only included in production planning problems but also in other areas such as the classical transportation problem as done by Damcı-Kurt et al. (2015). They consider a transportation problem where the suppliers have the choice of selecting the markets to satisfy their demands, which they refer to as transportation problem with market choice (TPMC). They prove that the TPMC is strongly NP-hard while the classical transportation problem is polynomially solvable. Aardal and Le Bodic (2014) provide polynomial-time reductions from TPMC to the capacitated facility location problem and give approximation algorithms. Walter et al. (2016) extend TPMC by adding a cardinality constraint that limits the number of markets that are rejected. They show that the cardinality constrained simple transportation problem with market choice is polynomially solvable.

The market selection decisions are also studied in disaster relief context. Kimiagari and Montreuil (2018) consider a market deployment planning problem for a natural disaster relief supply business venture, where it is crucial to determine the markets before designing and planning the activities, resources and financial flows. The decision is to choose in which period to deploy which market. They develop a multi-objective mixed-integer goal programming approach as a part of a solution method where the objectives are maximizing the expected present value from a time-phased market deployment and minimizing the goal deviation costs over a planning horizon.

Lot-sizing problems in a multi-objective setting are considered from different perspectives in the literature. Romeijn et al. (2014) consider a bi-objective economic lot-sizing problem where the objectives are minimizing the traditional lot-sizing costs over the whole planning horizon, and minimizing the maximum production and inventory expenses calculated over consecutive and disjoint blocks of time. The second objective is motivated by examples from limited physical capacities of scarce resources or from carbon emission limitations. They provide polynomially solvable cases for the Pareto efficient outcome problem in case of non-speculative lot-sizing costs. Mehdizadeh et al. (2016) study a multi-item capacitated lot-sizing problem with setup times and costs related with the demand shortage and safety stock deficit. They aim to minimize both the storage space required and the classical total cost. They propose two Pareto-based meta-heuristic algorithms. A similar study is considered by Ben Ammar et al. (2020) where demand backlogging is possible. They consider the inventory level as a separate objective function rather than including it in the total cost. They develop a particle swarm optimization to obtain a set of Pareto optimal solutions. There exist multiple studies that consider the lot-sizing problem with supplier selection in a multi-objective setting. Besides the classical lot-sizing total cost function, the objectives include the quality level of the products ordered (Rezaei and Davoodi 2011), total environmental score (Azadnia et al. 2015) and defect rate (Ustun and Demirtas 2008).

Application of decomposition-based algorithms in lot-sizing related problems is limited in the literature. Bahl and Zionts (1987) develop a Benders decomposition algorithm to solve a lot-sizing problem that minimizes the inventory and capacity costs where the subproblem becomes a transportation problem. Martinez et al. (2019) study an integrated process configuration, lot-sizing and scheduling problem where products can be produced by different process configurations. They propose a branch-and-check algorithm that uses logic-based Benders cuts. Bayley et al. (2018) consider a coordinated capacitated lot-sizing problem with multiple product families. They propose a solution algorithm that combines Benders decomposition with an evolutionary algorithm. They strengthen the Benders master problem by adding valid inequalities. However, Caserta and Voß (2021) show that one of the valid inequalities added to the master problem cuts off some feasible solutions, and hence, is not valid. Witthayapraphakorn and Charnsethikul (2019) use Benders decomposition to solve a lot-sizing problem with uncertain demand where the subproblem is solved with a special-purpose method. Gruson et al. (2021) study a three-level stochastic lot-sizing and replenishment problem that includes a distribution structure. They develop a Benders-based branch-and-cut algorithm to solve the problem by exploiting the network substructures identified in the decomposition.

We consider a lot-sizing problem with market selection in a single and multi-objective setting. The main contributions of our paper are: (i) it is the first time that an exact solution method based on a decomposition algorithm is proposed as a solution method for the integrated market selection and production planning problem, (ii) the decomposition algorithm is based on a game theoretical interpretation, which allows us to devise an efficient algorithm for solving the subproblem, (iii) we develop a multi-objective optimization model for a practically relevant lot-sizing problem with market selection and develop a three-phase algorithm that finds the complete Pareto frontier by incorporating our decomposition algorithm, (iv) our solution methods decrease the solution times substantially.

The remainder of this article is organized as follows. In Section 2, we formally define our problem. Section 3 provides our solution method that we developed for a single-objective problem. Then, Section 4 gives the multi-objective problem and our solution method to find the complete Pareto frontier. We give the results of our computational study in Section 5 and conclude the paper in Section 6.

2. Problem Description and Mathematical Model for Single-Objective Problem

We study an integrated market selection and single-item lot-sizing problem over a set of planning periods T, indexed by t. We consider a set of markets M, indexed by m, each of which has a deterministic demand, d_t^m, for each period $t \in T$. We have the option of either selecting a market to satisfy its demand throughout the planning horizon or rejecting the market and not including its demand in our planning. If we decide to satisfy the demand of the market throughout the planning horizon, then we obtain a certain revenue, R_m. There is a fixed set up cost, K_t, that we pay if we decide to produce in period $t \in T$, and a variable production cost, p_t, per every unit we produce. We pay an inventory holding cost of h_t for every unit that we put in the inventory at the end of period $t \in T$.

In order to formulate the problem, we introduce the following decision variables. The first set of decision variables are the market selection variables; z_m, which take the value of 1 if we decide to satisfy the demand of market $m \in M$; and 0, otherwise. Then, we have lot-sizing decision variables: y_t is a binary variable that takes the value of 1, if we decide to produce in period $t \in T$; and 0, otherwise. The variable x_{it}^m gives the amount of units produced in period $i \in T$ to satisfy the demand of market $m \in M$ in period $t \in T$.

Note that we use the well-known change of decision variables commonly used in the lot-sizing literature (see Krarup and Bilde (1977) who introduced this reformulation), leading to a stronger formulation compared to one using variables in which production quantities are aggregated. Let C_{it} represent the unit cost of producing in period $i \in T$ and holding cost from period i to period $t \in T$, i.e., $C_{it} = p_i + \sum_{j=i}^{t-1} h_j$. Then, we can write our mathematical model as follows:

$$\text{(MIP)} \quad \max \quad \sum_{m\in M} R_m z_m - \sum_{t\in T}\left(K_t y_t + \sum_{i\le t}\sum_{m\in M} C_{it} x_{it}^m\right) \tag{1}$$

$$\text{s.t.} \quad \sum_{i\le t} x_{it}^m = d_t^m z_m, \qquad t\in T; m\in M \tag{2}$$

$$x_{it}^m \le d_t^m y_i, \qquad i,t\in T; i\le t; m\in M \tag{3}$$

$$x_{it}^m \ge 0, \qquad i\le t\in T; m\in M \tag{4}$$

$$y_i, z_m \in \{0,1\}, \qquad i\in T; m\in M. \tag{5}$$

The objective function (1) maximizes the net profit that is obtained by subtracting the lot-sizing cost from the revenue obtained from the markets that are selected. Constraints (2) are demand satisfaction constraints for the markets that we select throughout the planning horizon. Constraints (3) make sure that we can produce only if we pay the setup cost and the amount of production for period t does not exceed the demand at that period. Constraints (4) and (5) are the nonnegativity and binary restrictions for the decision variables, respectively.

3. Solution method for Single-objective Market Selection

In this section we provide our solution method, which is based on a decomposition algorithm. First, we provide a solution approach to model the subproblem that is related to cooperative game theory. Then, we propose algorithmic improvements on the decomposition method.

We propose a decomposition-based cutting plane algorithm to solve problem (MIP). The main idea is to decompose the problem into a master problem and a subproblem where the master problem concerns the markets to be selected while the subproblem takes care of the lot-sizing cost. With this in mind, the master problem can be written as

$$\text{(MP)} \quad \max \quad \sum_{m\in M} R_m z_m - \theta \tag{6}$$

$$\text{s.t.} \quad \theta = \nu(\mathbf{z}), \tag{7}$$

$$z_m \in \{0,1\}, \quad m \in M, \tag{8}$$

where $\nu(\mathbf{z})$ represents the lot-sizing cost for market selection $\mathbf{z} = (z_1, \ldots, z_{|M|})$.

For a fixed set of market selection variables $\tilde{\mathbf{z}}$, the subproblem can be formulated as:

$$(\text{SP}) \quad \nu(\tilde{\mathbf{z}}) = \min \quad \sum_{t \in T} \left(K_t y_t + \sum_{i \le t} \sum_{m \in M} C_{it} x_{it}^m \right) \tag{9}$$

$$\text{s.t.} \quad \sum_{i \le t} x_{it}^m = d_t^m \tilde{z}_m, \qquad t \in T; m \in M \tag{10}$$

$$x_{it}^m \le d_t^m y_i, \qquad i \le t \in T; m \in M \tag{11}$$

$$x_{it}^m \ge 0, \qquad i \le t \in T; m \in M \tag{12}$$

$$y_i \in \{0,1\}, \qquad i \in T; m \in M. \tag{13}$$

The aim is to formulate (7) as a set of linear inequalities or cuts in the z-variables that represent the subproblem's objective value in a valid way for a given market selection. That is, such a cut should (i) represent the lot-sizing cost for the given market selection, and (ii) not overestimate the cost of any other market selection. To derive these cuts, we take an approach based on economic lot-sizing games (Van den Heuvel et al. (2007), Chen and Zhang (2016)), which is in the field of cooperative game theory.

3.1. Cooperative game theory - Economic Lot-sizing games

We introduce the economic lot-sizing games in this section. To show the relation between the game and the market selection problem, we reuse some notation (instead of introducing more notation). Let us consider a set of retailers, M, that sell the same item to the market where all the demand over a set of planning period, T, are known to them. Suppose that there is a single manufacturer from which all retailers buy the items, and this manufacturer charges ordering cost for each order and a production cost for units ordered, which is linear.

Retailers may order from the manufacturer at amounts to cover a single or more periods' demand. A holding cost is incurred per item put in the inventory at the end of a period, which is linear and equal for all retailers. In this setting, if a subset of retailers cooperates to place a joint order instead of individual orders, a cost saving will be obtained in general. Let us formally define the problem:

Definition 1. Let $\mathbf{d}^i = (d^i_1, \ldots, d^i_{|T|})$ represent retailer i's demand vector and $\mathbf{d}^N$ represent the demand of coalition $N \subseteq M$ (a subset of retailers that cooperates), i.e., $\mathbf{d}^N = \sum_{i \in N} \mathbf{d}^i$. The optimal procurement plan for coalition N can be defined as a lot-sizing problem with demand $\mathbf{d}^N$ where the cost of this procurement plan is given as $C^{LS}(N)$.

Now, the costs $C^{LS}(N)$ need to be allocated to the retailers in a 'fair' way such that (i) all costs are divided among the retailers in the coalition N, and (ii) no subsets of markets, i.e. the coalition $S \subseteq N$, should be worse off compared to only cooperating with retailers in set S, and hence have an incentive to stop the cooperation. With β_r denoting the cost allocated to retailer r, this translates to

$$\sum_{r \in N} \beta_r = C^{LS}(N), \text{ and} \tag{14}$$

$$\sum_{r \in S} \beta_r \leq C^{LS}(S) \text{ for all } S \subseteq N. \tag{15}$$

Equation (14), known as the efficiency constraint in cooperative game theory, ensures that all costs are divided among retailers, while Equation (15), known as the rationality constraints, makes sure that no subcoalition S is worse off.

Definition 2. As a well-known concept in cooperative game theory, all cost allocations satisfying (14) and (15) are called the core of the game, denoted by

$$core(N) = \left\{ \boldsymbol{\beta} \in \mathbb{R}^{|N|} : \sum_{r \in N} \beta_r = C^{LS}(N) \text{ and } \sum_{r \in S} \beta_r \leq C^{LS}(S) \text{ for all } S \subseteq N \right\}. \tag{16}$$

3.2. Formulating the lot-sizing costs

It turns out that the core elements in the economic lot-sizing games can be used to formulate the lot-sizing cost in a valid way in terms of the z variables, and hence, form the basis of our decomposition algorithm.

PROPOSITION 1. *Let $\boldsymbol{\beta}^N \in core(N)$ be the cost allocation vector for some subset of markets $N \subseteq M$. Then constraint (7) in (MP) can be replaced by cuts*

$$\theta \geq \sum_{m \in N} \beta_m^N z_m \text{ for all } N \subseteq M. \tag{17}$$

Proof: In order to prove the proposition, we need to show that for a given market selection N, the term $\sum_{m \in N} \beta_m^N z_m$ (i) represents the optimal lot-sizing cost $C^{LS}(N)$ corresponding to this market selection N, and (ii) it is valid for another market selection $N' \subseteq M$, that is, it does not overestimate the cost $C^{LS}(N')$ and potentially cut off an optimal solution.

By definition, the variables $\mathbf{z}$ representing market selection N satisfy $z_m = 1$ if and only if $m \in N$. Using the efficiency constraints (14) of $core(N)$ we have

$$\sum_{m \in N} \beta_m^N z_m = \sum_{m \in N} \beta_m^N = C^{LS}(N). \tag{18}$$

Now consider another market selection N' represented by the variables $\mathbf{z}'$ with $z'_m = 1$ if and only if $m \in N'$. It follows that

$$\sum_{m \in N} \beta_m^N z'_m = \sum_{m \in N' \cap N} \beta_m^N \leq C^{LS}(N' \cap N) \leq C^{LS}(N'), \tag{19}$$

where the first inequality follows from the rationality constraints (15) of $core(N)$ applied to $S = N' \cap N$, and the second inequality from the fact that satisfying more markets cannot lead to lower lot-sizing cost. □

Lifting cut set (17)

Note that no β_m^N coefficients appear for markets $m \notin N$ in constraints (17), and hence no z_m variables appear for $m \in M \backslash N$. Since θ in (MP) represents the lot-sizing cost, we can add the z_m variables ($m \in M \backslash N$) with a positive coefficient, taking into account that at least some variable cost are incurred. In order to specify these coefficients, define VC_m as the lowest possible variable cost of satisfying market m's demand among any possible solution, computed by solving a lot-sizing problem only serving demand $\mathbf{d}^m$ and assuming that $K_t = 0$. Clearly, this constitutes a valid lower bound on the cost of serving market m. Using this result, we can lift (17), which results in Proposition 2. When analyzing the proof, we are essentially exploiting the slack in the last inequality of (19) to do the lifting.

PROPOSITION 2. *Replacing constraints* (17) *by*

$$\theta \geq \sum_{m \in N} \beta_m^N z_m + \sum_{m \in M \backslash N} VC_m z_m \text{ for all } N \subseteq M \tag{20}$$

leads to a stronger and still valid formulation.

Proof: Using the lifted cut, it is clear that the constraints are stronger as the markets $m \notin N$ have non-negative coefficients. To show that they are valid for any market selection N', represented by the variables $\mathbf{z}'$ with $z'_m = 1$ if and only if $m \in N'$, it follows that

$$\sum_{m \in N} \beta_m^N z_m + \sum_{m \in M \backslash N} VC_m z_m = \sum_{m \in N' \cap N} \beta_m^N + \sum_{m \in N' \backslash N} VC_m \leq C^{LS}(N' \cap N) + \sum_{m \in N' \backslash N} VC_m \leq C^{LS}(N'), \tag{21}$$

where the last inequality follows from the fact that for serving the additional markets $N' \backslash N$ on top of markets $N' \cap N$, there will be a cost increase of at least the lowest possible variable cost to serve these additional markets. □

3.3. Decomposition Algorithm

We are ready to present the decomposition algorithm. Let θ represent the lot-sizing cost for selected markets. Then, using Proposition 2, the master problem can be formulated as

$$\text{(MP2)} \quad \max \quad \sum_{m\in M} R_m z_m - \theta \tag{22}$$

$$\text{s.t.} \quad \theta \geq \sum_{m\in N} \beta_m^N z_m + \sum_{m\in M\setminus N} VC_m z_m \text{ for all } N \subseteq M, \tag{23}$$

$$z_m \in \{0,1\}, \quad m \in M \tag{24}$$

$$\theta \geq 0. \tag{25}$$

The issues with this formulation are that (i) there is an exponential number of constraints (23), and (ii) we need a procedure to find the β_m^N coefficients in (23). To deal with issue (i), instead of pre-computing all constraints (23) upfront, we generate them in a cutting plane fashion. This leads to the algorithm given in Algorithm 1.

Algorithm 1 Decomposition Algorithm

Initialization:

Set of cuts, $C \leftarrow \emptyset$

Solve (MP2) with cut set C, resulting in an optimal solution $(\tilde{\mathbf{z}}, \tilde{\theta})$

while $\tilde{\theta} < \nu(\tilde{\mathbf{z}})$ **do**

  Generate cut of type (23)

  Add newly generated cut to set C

  Solve (MP2) with cut set C, resulting in an optimal solution $(\tilde{\mathbf{z}}, \tilde{\theta})$

end while

Output the optimal solution

Issue (ii) can be dealt with by using results from the literature on economic lot-sizing games. For a given market selection $\tilde{\mathbf{z}}$ corresponding to markets $N = \{m \in M : \tilde{z}_m = 1\}$, finding the coefficients β_m^N in (23) is equivalent to finding an element $\boldsymbol{\beta}^N \in core(N)$ as given in (16). Chen and Zhang (2016) study the economic lot-sizing game in detail and show how to efficiently find $\boldsymbol{\beta}^N \in core(N)$ by using the duals of the simple plant location formulation (see Theorem 1, p. 1206). To be precise, let μ_t be the dual variables of the demand satisfaction constraints of this formulation (i.e., first constraint set of formulation (2) in Chen and Zhang (2016)) with the demands set to $d_t = \sum_{m\in N} d_t^m = \sum_{m\in M} \tilde{z}_m d_t^m$ for $t = 1,\dots,T$. Note that these constraints correspond to constraints (10) of our subproblem (SP) but aggregated over the markets. Then the β_m^N coefficients required in (23) should be set to

$$\beta_m^N = \sum_{t=1}^{T} \mu_t d_t^m \text{ for } m \in N. \tag{26}$$

In turn, this dual formulation is analyzed in detail by Wagelmans et al. (1992). They show that the dual variables μ_t can be calculated efficiently by a recursive scheme instead of solving an LP.

Remark 1

Formulation (MIP) contains $(M+T)$ binary variables and $\mathcal{O}(MT^2)$ continuous variables. On the other hand, the master problem of our decomposition algorithm (MP2) contains M binary variables and a single continuous variable. As explained above, we solve the subproblem using the algorithm of Wagelmans et al. (1992) and calculate dual variables without explicitly solving an optimization model. Thus, the total number of variables is significantly reduced in our decomposition approach. On the other hand, (MIP) contains a polynomial number of constraints whereas (MP2) has an exponential number of constraints, which our decomposition algorithm generates dynamically in a cutting-plane fashion.

Remark 2

Note that (MIP) can be solved using Benders decomposition. In the classical Benders decomposition, integer variables (i.e., y and z variables) constitute the master problem, where the continuous x variables and the corresponding constraints are handled in the subproblem. Van den Heuvel et al. (2012) show that it is possible to relax either the y or z variables in (MIP) as continuous variables, and prove that the relaxed variables will take on integer values in an optimal solution. This property suggests an alternative Benders decomposition strategy, where we relax the y variables and only keep z variables as binary. In this approach, the z variables and the corresponding objective function term constitute the master problem, which is similar to our decomposition approach. The x, y variables and Constraints (2)–(4) constitute the subproblem, which is solved as a linear programming problem to generate Benders cuts. Note that due to the interpretation based on economic lot-sizing games, we are able to lift the cuts (see Proposition 2 and Equation (20)), something that will be absent in a classical Benders decomposition approach. We test the efficacy of this approach and compare it with our decomposition algorithm in Section 5.

3.4. Algorithmic Improvements

We provide some algorithmic improvements that will be applied on the decomposition algorithm in this section.

3.4.1. Preprocessing Let TC_m denote the optimal total cost of serving market m individually, and let VC_m denote the lowest possible variable cost of serving market m. (Note that $TC_m = C^{LS}(\{m\})$, but we prefer to introduce a separate notation for clarity.) We calculate TC_m by running any lot-sizing algorithm (in our case the algorithm of Wagelmans et al. (1992)) for market m considering demand d_t^m, setup cost K_t, production cost p_t and inventory holding cost h_t. Similarly, we calculate VC_m for market m assuming that $K_t = 0$. The following are valid in an optimal solution:

$$z_m = 1 \quad m \in M : R_m \geq TC_m, \tag{27}$$

$$z_m = 0 \quad m \in M : R_m < VC_m. \tag{28}$$

Preprocessing rule (27) is based on the observation that if the revenue of a market is larger than the total cost (setup + variable cost) of serving that market by itself, then it will be selected in an optimal solution. On the other hand, (28) implies that if the revenue of a market is smaller than the variable cost of serving that market, then it cannot be selected in an optimal solution.

3.4.2. Valid Inequalities In the initial solution of the master problem (MP2) all $z_m = 1$ and $\theta = 0$, which yields a weak upper bound on the optimal objective function value. We can add some valid inequalities to obtain a tighter upper bound. We first observe that θ cannot be less than the minimum total cost of a selected market m plus the summation of the lowest possible variable cost of satisfying the demand of each other selected market, that is,

$$\theta \geq TC_m z_m + \sum_{m' \in M, m' \neq m} VC_{m'} z_{m'}, \quad m \in M. \tag{29}$$

For our second valid inequality, let TC denote the total cost of satisfying demand of all markets, which we calculate by considering $d_t = \sum_{m \in M} d_t^m$, setup cost K_t, production cost p_t and inventory holding cost h_t. Then, the following is valid:

$$\theta \geq TC - \sum_{m \in M} TC_m (1 - z_m). \tag{30}$$

Valid inequality (30) is based on our observation that in earlier iterations the master problem tends to set most z_m values to 1 while significantly underestimating the cost θ. This inequality ensures that θ correctly estimates the true cost if all markets are selected, and yields a valid bound for other market selections.

To derive the last set of valid inequalities, recall from (27) that any market having $R_m \geq TC_m$ must be selected in an optimal solution. However, for any market m having $R_m < TC_m$, if m is selected then a number of other markets must also be selected to compensate for the loss, leading to the valid inequality

$$\sum_{m' \in M, m' \neq m} (R_{m'} - VC_{m'}) z_{m'} \geq (TC_m - R_m) z_m, \quad m \in M \text{ with } R_m < TC_m. \tag{31}$$

Building on this idea, we can generalize (31) as

$$\sum_{m' \in M, m' \neq m} (R_{m'} - VC_{m'}) z_{m'} + (R_m - TC_m) z_m \geq LB \quad m \in M, \tag{32}$$

where LB denotes the objective function value of a known feasible solution. Note that the cost terms at the left hand side are a lower bound on the lot-sizing cost as in (29), showing the validity of the inequality. In our case we use the heuristic of Van den Heuvel et al. (2012) as a lower bound.

4. Multi-objective Market Selection Problem

In this section, we consider the market selection problem in a multi-objective setting. We start with providing the motivation for studying the integrated market selection problem in this setting. Then, we give our solution approach, which we call a three-phase method, that is able to compute the complete Pareto frontier.

4.1. Motivation

While the main goal of a company is to maximize profit, companies often try to increase their market share for strategic reasons at the expense of sacrificing some profit. Hence a company could be interested in the trade-off between revenue and cost. Let z_1 (resp. z_2) be the first (resp. second) objective. Then we can model this trade-off as a multi-objective model where the objectives are

$$(z_1, z_2) = \left(\sum_{m \in M} R_m z_m, \sum_{t \in T} \left(K_t y_t + \sum_{i \leq t} \sum_{m \in M} C_{it} X_{it}^m \right) \right). \tag{33}$$

Another motivation for the multi-objective setting emerges from the economics area, where it is important to calculate the return on investment (ROI) before committing to an investment opportunity, where ROI is defined as the ratio of the net return of an investment and the cost of the investment. In our market selection setting, this corresponds to maximizing the objective

$$\frac{\sum_{m\in M} R_m z_m - \sum_{t\in T}\left(K_t y_t + \sum_{i\le t}\sum_{m\in M} C_{it} X_{it}^m\right)}{\sum_{t\in T}\left(K_t y_t + \sum_{i\le t}\sum_{m\in M} C_{it} X_{it}^m\right)}. \tag{34}$$

Although this objective function makes the formulation nonlinear, Megiddo (1979) shows that this type of problem can be solved as a parametric optimization problem, which in turn can be considered as a weighted multi-objective problem. In other words, the market selection problem with maximizing ROI can be tackled by solving a "Net revenue vs. cost" multi-objective problem.

Besides the more practical motivation, there is also a technical reason why we consider the multi-objective setting of the problem. Note that points which are close to each other on the Pareto frontier correspond to solutions of optimization problems that are slightly different from each other. This suggests that reusing cuts could help to warm start the problem of finding a new Pareto point on the frontier, and hence, reusing cuts could help in determining the entire Pareto frontier in an efficient way. In the next section we propose an approach where this idea is exploited.

4.2. Solution approach

A common approach to solve a multi-objective problem is to use the so-called weighted method, where the different objectives are weighted and transformed into a single objective. A disadvantage of this method is that only the extreme supported Pareto points are found. An alternative is to use the well-known ε-constraint approach, where one objective is bounded by ε and added as a constraint, while the other objective is optimized. The possible number of values of ε, and hence, the optimization problems to be solved can be huge, which is a disadvantage of this approach. Therefore, an alternative is to use a two-phase method, where in the first phase one starts with a partial Pareto frontier (typically consisting of the extreme supported points), while in the second phase one searches for additional points between points on the partial frontier. For an overview of approaches to solve multi-objective problems we refer the interested reader to Ehrgott (2005). Inspired by the two-phase method, we propose a three-phase algorithm tailored to our problem where the relevant values of the second objective are computed first.

The general idea of our three-phase algorithm to compute the entire Pareto frontier is as follows. In Phase I we create a set of basis points containing the relevant objective values, i.e., the possible revenue amounts and their corresponding lot-sizing costs, in Phase II we perform a dominance check to get an approximate frontier, and finally in Phase III we check for optimality. To formalize our approach, we represent a point on the Pareto frontier as a tuple $\mathbf{p} = (Rev_p, C_p, N_p)$, where N_p is the market selection, with the corresponding revenue $Rev_p = \sum_{m \in N_p} R_m$ and the lot-sizing cost $C_p = C^{LS}(N_p)$. The Pareto frontier consists of all non-dominated points, where a point $\mathbf{q}$ is dominated by $\mathbf{p}$ if $Rev_q \leq Rev_p$ and $C_q \geq C_p$ where strict inequality holds in one of the two. We now describe the three phases in more detail.

Phase I:

In this phase we construct points (possibly including dominated ones) such that all possible revenues considering all markets are attained. We denote the set of all attainable revenues by R, which is formally defined as $R = \bigcup_{N \subseteq M} \left\{ \sum_{m \in N} R_m \right\}$, and the corresponding Pareto points by P. We can obtain these sets by starting from an empty market selection, and iteratively extending the current market selection by a single market. The details of this phase are found in Algorithm 2.

Algorithm 2 Phase I of three-phase algorithm

Create initial $\mathbf{p} := (0, 0, \emptyset)$

Set $P \leftarrow \{\mathbf{p}\}$ (the set of Pareto points)

Set $R \leftarrow \{Rev_p\}$ (the set of attained revenues)

for each $m \in M$ **do**

  while there is a point $\mathbf{p} \in P$ which has not been extended by m **do**

    Extend $\mathbf{q}$ from $\mathbf{p}$ with $N_q \leftarrow N_p \cup \{m\}$, $Rev_q = \sum_{m \in N_q} R_m$ and $C_q = C^{LS}(N_q)$

    If $Rev_q \in R$, then update cost of corresponding point (if it has improved)

    If $Rev_q \notin R$, then $P \leftarrow P \cup \{\mathbf{q}\}$ and $R \leftarrow R \cup \{Rev_q\}$

  end while

end for

Phase II:

Remove all dominated points from P, which can be done efficiently once the points are sorted by one of the objectives. After this phase we have an approximate Pareto frontier.

Phase III:

We now check the approximate Pareto frontier for optimality. Let $\mathbf{p}$ and $\mathbf{q}$ be two consecutive points on the approximate frontier, i.e., there is no other point with revenue value in the interval $\langle Rev_p, Rev_q \rangle$. We solve a market selection problem ($\text{MO}_{p,q}$) where the revenue is bounded in $\langle Rev_p, Rev_q]$ while minimizing the cost. The problem ($\text{MO}_{p,q}$) can be formulated as

$$(\text{MO}_{p,q}) \qquad \min \quad \theta \tag{35}$$

$$\text{s.t.} \quad Rev_p < \sum_{m\in M} R_m z_m \leq Rev_q \tag{36}$$

$$\theta \geq \sum_{m\in N} \beta_m^N z_m + \sum_{m\in M\setminus N} VC_m z_m \text{ for all } N \subseteq M \tag{37}$$

$$z_m \in \{0,1\}, \quad m \in M, \tag{38}$$

$$\theta \geq 0. \tag{39}$$

where the strict inequality in (36) can be replaced by $Rev_p + 1 \leq \sum_{m\in M} R_m z_m$ in case of integer data. There are three possible results we may get after solving (MO$_{p,q}$):

1. We find no solution with cost lower than C_q, which means the approximate frontier is optimal on the interval $\langle Rev_p, Rev_q]$.

2. We find a point $\mathbf{r}$ with cost $C_r < C_q$ and $Rev_r = Rev_q$. We can now replace $\mathbf{q}$ by $\mathbf{r}$ and there can be no other Pareto points in $\langle Rev_p, Rev_q]$.

3. We find a point $\mathbf{r}$ with cost $C_r < C_q$ and $Rev_r < Rev_q$. We add point $\mathbf{r}$ to the frontier and perform a dominance check. We need to solve (MO$_{r,q}$) on $\langle Rev_r, Rev_q]$ in search for further Pareto points.

Note that problem (MO$_{p,q}$) can be solved using our decomposition approach of Section 3, where constraints (37) are generated in a cutting plane fashion. Suppose that we have three consecutive points $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ where we solve (MO$_{p,q}$) and (MO$_{q,r}$) over the intervals $\langle Rev_p, Rev_q]$ and $\langle Rev_q, Rev_r]$. As the problems are similar, it is likely that common cuts will be generated. This suggests that when one problem is solved, we can 'warm start' the next problem by including the cuts that have been generated so far. Note that (MO$_{p,q}$) can be strengthened by adding valid inequalities (29)–(30) of Section 3.4.2. This does not hold for the pre-processing step and inequalities (31) and (32), as the Pareto frontier may contain solutions that are not profitable, i.e., the cost is higher than the revenue. A pseudocode of Phase III is provided in Algorithm 3. Note that Algorithm 3 focuses on the optimization problems that need to be solved. Points that are verified to be Pareto optimal are stored in a separate list and not shown in the pseudocode. The effectiveness of our three-phase approach including the reuse of cuts is tested in Section 5.2.

Algorithm 3 Phase III of three-phase algorithm

Input: sorted list of points P of approximate Pareto frontier

Set $\mathcal{C} \leftarrow \emptyset$ (set of generated cuts)

while $|P| \geq 2$ **do**

  Let $\mathbf{p}$ and $\mathbf{q}$ be the last two points in P

  Solve (MO$_{p,q}$) with set of cuts $\mathcal{C}$

  Let C^* be optimal objective value with market selection $\mathbf{z}^*$ and revenue Rev^*

  if $C^* < C^q$ and $Rev^* < Rev_q$ **then**

    Add new point $\mathbf{r} = (Rev^*, C^*)$ to P

  else if $C^* < C^q$ and $Rev^* = Rev_q$ **then**

    Update $C_q \leftarrow C^*$

  end if

  Remove $\mathbf{q}$ from list P

  Update $\mathcal{C}$ with newly generated cuts

end while

5. Computational Study

We provide the results of a computational study in which we compare the performance of the solution algorithms provided for the single and multiple objective problem with the direct solution of the problems (MIP) using a commercial solver. We use the data set provided in (Van den Heuvel et al. 2012) except for a modification of the production cost, which were equal to zero in the original data. In our instances we have set the production cost in such a way that it comprises a third of the total cost in case all markets are selected (next to a third of setup cost and holding cost), and the average revenues per item are increased by the production cost. A summary of the parameter values can be found in Table 1, where $\bar{d} = 5$ is the average demand, $n = \sqrt{2\alpha}$ is the expected time between orders and α controls this value. For more details on the data generation process we refer to Van den Heuvel et al. (2012).

Table 1 Parameters settings

parameter	d_t	K_t	p_t	h_t	R_m
value	Unif$[0, 2\bar{d}]$	$\alpha M \bar{d}$	$(2n-1)2/3$	1	$T\bar{d}((2n-1)/2 + p_t)$

We implemented all formulations and solution algorithms in C++, and performed all tests on a computer with Intel Xeon Silver 4214R 2.4 GHz CPU, 64 GB RAM running Windows Server 2019 operating system. We used CPLEX 20.1 as the solver for all implementations, where we used default solver settings except limiting the maximum number of threads to 4. We enforce a time limit of 1200 seconds for all instances.

5.1. Single-Objective Problem Computational Analysis

We provide the performance of the MIP formulation, the automatic Benders algorithm of CPLEX (BA) and the decomposition algorithm (DA) that we propose to solve the single-objective problem in this section.

First, we test the performance of MIP, BA and DA without any additional improvements given in Section 3.4, and provide the corresponding results in Table 2. Each problem set, given as a row in Table 2, includes ten problem instances that are created using different number of markets, time periods and α values. For each of these problem sets, we report the following statistics, calculated over ten random instances: “Time”: The average amount of time in seconds spent by each algorithm, "Gap": The average final percentage optimality gap, and "#Cut": The total number of cuts generated by the algorithm over ten instances. We provide the averages over all problem sets at the bottom line of the table.

Table 2 Summary for Single-objective Problem

$\|M\|$	$\|T\|$	α	MIP		BA		DA		
			Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	#Cut
40	40	2	5.6	0	11.9	0	0.3	0	3542
		5	6.6	0	27.8	0	0.5	0	4474
		8	10.4	0	67.4	0	1.7	0	7579
		11	6.2	0	31.0	0	0.2	0	3074
40	80	2	131.2	0	237.0	0	2.2	0	9716
		5	101.0	0	215.2	0	0.4	0	4327
		8	90.7	0	280.9	0	0.3	0	3452
		11	166.4	0	611.9	0	2.4	0	8510
		Avg	64.8	0	185.4	0	1.0	0	5584

As we see in Table 2, all instances are solved to optimality within the given time limit for all settings. For MIP and BA, as the number of time periods increases, it takes considerably longer to solve the instances. In terms of CPU time, MIP performs consistently better than BA on all settings with an average of 64.8 seconds over all instances while it takes 185.4 seconds for BA. When we analyze our solution algorithm DA, we see that all instances are solved to optimality within a few seconds with an average of 1.0 seconds. The number of cuts generated by the algorithm is insensitive to the number of periods. Overall, we can conclude that our proposed solution algorithm improves the solution performance substantially.

Next, we test the effect of the algorithmic improvements that we introduce in Section 3.4. In order to see the individual effect of each improvement, we add each separately to our decomposition algorithm and provide the results for five different settings where "PP" represents the addition of (preprocessing) inequalities (27) and (28), and the valid inequalities that are given with their equation numbers in Table 3.

Table 3 Effect of Algorithmic Improvements on the Decomposition Algorithm

$\|M\|$	$\|T\|$	α	PP		VI(29)		VI(30)		VI(32)	
			Time (s)	#Cut	Time (s)	#Cut	Time (s)	#Cut	Time (s)	#Cut
40	40	2	0.3	3353	0.3	3354	0.3	3531	0.3	3630
		5	0.5	4382	0.5	4264	0.6	4455	0.5	4284
		8	1.4	7684	1.4	7545	1.8	7654	1.4	7571
		11	0.2	2965	0.2	3069	0.2	2973	0.2	3150
40	80	2	2.0	9959	2.1	9347	2.8	10123	2.1	9581
		5	0.4	4296	0.4	4194	0.4	4322	0.4	4179
		8	0.3	3504	0.4	3557	0.3	3359	0.4	3489
		11	1.8	8125	1.7	8163	2.0	8253	1.8	8088
		Avg	0.9	5534	0.9	5437	1.0	5584	0.9	5497

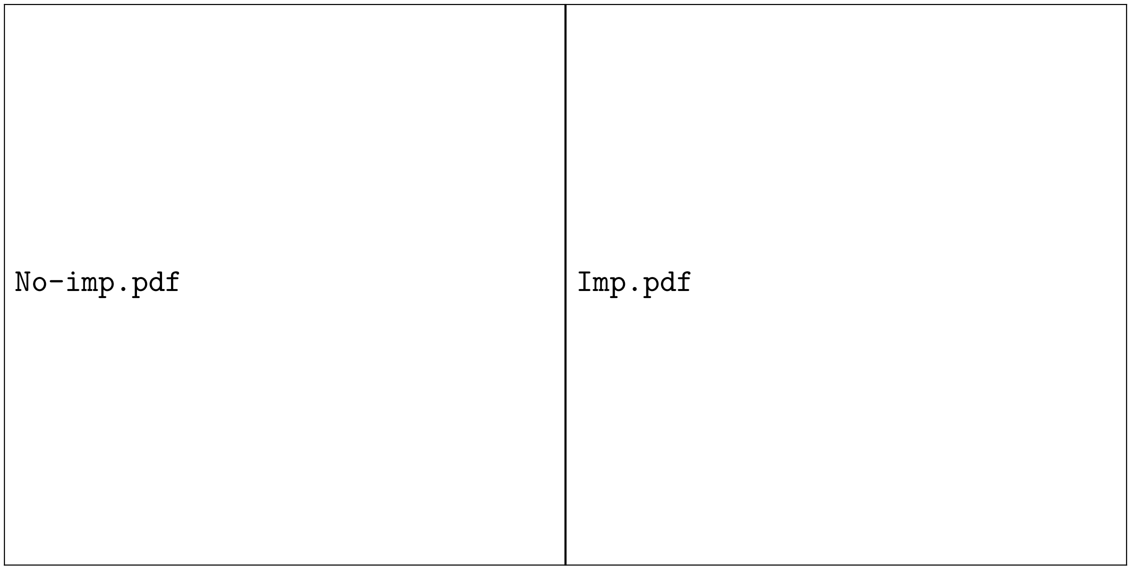
As we see in Table 3, the addition of valid inequalities has incremental effect, where all valid inequalities help the decomposition algorithm to find an optimal solution faster except inequality (30). Therefore, we omit this valid inequality for the other computational tests.

We also analyse the effect of these improvements on the MIP and BA. Hence, we add all effective algorithmic improvements to the MIP ("Imp-MIP"), Benders algorithm ("Imp-BA") and our decomposition algorithm ("Imp-DA"), and provide the results in Table 4. As seen in Table 4, the preprocessing rules and valid inequalities help MIP converge to optimality faster within an average of 32.0 seconds, which means the average solution time is now 50% lower. When we analyze the BA with improvements, while the average time to solve all instances decreases to 106.9 seconds, one of the instances is not solved to optimality within the given time limit. Finally, when we add all effective improvements to DA, we see slightly better results within an average solution time of 0.8 seconds.

Table 4 Comparison of Our Algorithm with MIP and Benders Algorithm

$\|M\|$	$\|T\|$	α	Imp–MIP	Imp–BA			Imp–DA	
			Time (s)	Time (s)	Gap (%)	#Sol	Time (s)	#Cut
40	40	2	2.8	7.6	0	10	0.2	2973
		5	3.8	17.5	0	10	0.5	3987
		8	6.3	44.4	0	10	1.4	7284
		11	4.0	21.0	0	10	0.1	2753
40	80	2	70.3	123.7	0	10	1.8	9301
		5	44.4	111.3	0	10	0.4	3872
		8	47.7	160.4	0	10	0.2	2869
		11	76.95	369.3	6.7	9	1.7	7741
		Avg	32.0	106.9	0.8	9.9	0.8	5098

We visualize the computational results by using a performance profile chart (Dolan and Moré 2002) for the set where $M = 40$ and $T = 40$. We use CPU time as the performance metric. Figure 1 shows $P_s(\tau)$, which is the percentage of times that algorithm s can find optimal solution within factor τ of the minimum solution time among all algorithms. For example, if we choose $\tau = 100$, the MIP method finds an optimal solution with a CPU time within a factor of 100 of the minimum solution time in about 75% of the problem instances while this value is around 25% for BA. Note that $P_s(1)$ shows the percentage of times that algorithm s can find the optimal solution with minimum CPU value over all problem sets. Also note that the solution times are not aggregated for different computational settings, which means that we take all 4×10 problem instances' solution times.



(a) Basic Algorithms (b) Algorithms with Improvement

Figure 1 Performance Profile Chart

The trend is similar for both basic algorithms and the improved ones, where DA and Imp-DA significantly outperform the other algorithms for all instances. The MIP can solve all instances with a CPU time within a factor of approximately 150 of the minimum solution time while this value is 450 for BA. These values get smaller when we use the improved algorithms. For Imp-MIP and Imp-BA, all instances are solved with a CPU time within a factor of approximately 100 and 300 of the minimum solution time, respectively.

5.2. Multi-Objective Problem Computational Analysis

Before we present the efficacy of our solution approach for the multi-objective setting, we first show how a typical multi-objective solution looks like. Figure 2 shows the revenue and cost objective function values and the complete Pareto frontier with the proposed solution algorithm obtained for one of the problem instances in our test suite. As expected (0, 0) is a Pareto point in which none of the markets is selected. Similarly, selecting all markets also yields a Pareto efficient solution having maximum revenue and maximum cost. The red dot in the figure shows the optimal single objective solution maximizing the profit. Note that while the set of extreme supported Pareto points results in a convex curve, since our algorithm calculates complete Pareto frontier, the curve is not convex. As illustrated in Figure 2, our algorithm can help decision makers better understand the tradeoff between the additional revenue generated by increasing selected markets and the corresponding operating costs.

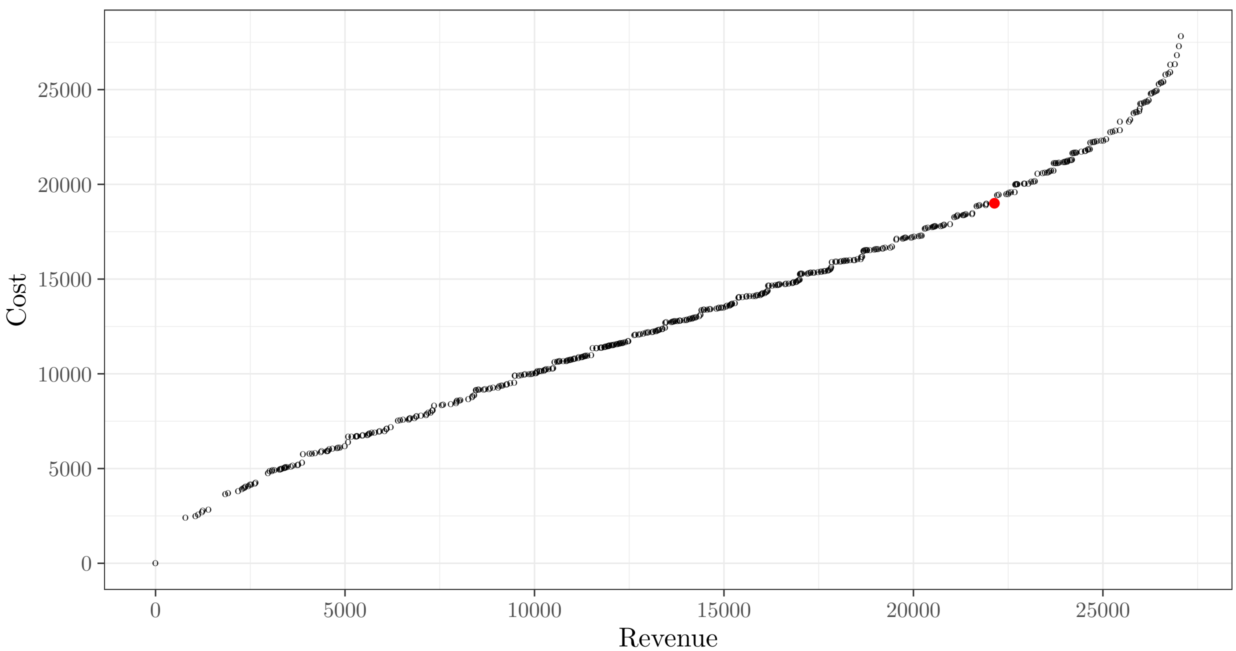


Figure 2 Revenue vs Cost for a Problem Instance

We now discuss the computational results for the multi-objective setting. We generate data according to the same setting given in the single-objective part except that we have different number of markets, which are 10, 20, 30 and 40, and a single setting for the time period, which is 40. Since the algorithm does not give acceptable results within a given time limit, we omit the results for the ones that have 80 time periods.

We report the results for different phases separately in Table 5, where for every row we have 10 problem instances. In this table "Time" refers to the average amount of CPU time in seconds per instance; "#points" refers to the total number of revenue-cost pairs (possibly Pareto points) generated in Phase I; "#eff" refers to the total number of Pareto points found in Phase II over ten instances; "%new eff" refers to the percentage of new Pareto points found in Phase III; and finally "%imp" refers to the percentage of solutions improved in Phase III (i.e., having the same revenue but less cost). Since the time required for Phase II is in the order of milliseconds, we do not report it. Furthermore, the CPU time of Phase III will be discussed in detail in Table 6 as there are two ways of performing this phase.

Table 5 Summary for Multi-objective Problem

M	T	α	Phase I		Phase II	Phase III	
			Time (s)	#points	#eff	#new eff (%)	#imp(%)
10	40	2	0.1	9167	332	0.00	0.00
		5	0.1	9780	384	0.00	0.00
		8	0.1	9738	328	0.00	0.00
		11	0.1	9830	313	0.00	0.00
20	40	2	5.5	117898	1142	0.00	0.09
		5	8.8	197826	1246	0.00	0.00
		8	10.7	256660	1206	0.00	0.00
		11	11.9	299606	1210	0.08	0.00
30	40	2	19.2	199700	2590	0.08	0.04
		5	33.0	348890	2510	0.15	0.00
		8	41.9	435434	2703	0.09	0.06
		11	50.2	543976	2778	0.00	0.00
40	40	2	39.7	269894	4450	0.06	0.11
		5	70.9	481981	4766	0.03	0.00
		8	89.4	424177	3260	0.26	0.14
		11	110.4	771142	5031	0.12	0.00
		Avg	30.8	274106	2141	0.05	0.03

As we see in Table 5, the required time to solve Phase I is less than one second for small problem sizes. As the problem size increases, the required time also increases, where the average time for all instances is 30.8 seconds. We can see the same trend for the number of potential Pareto points generated with an average of 274,106 over all batches of 10 instances. The number of Pareto points found in Phase II also increases with the problem size in line with the number of points generated in Phase I. In Phase III, we solve $(\text{MO}_{p,q})$ over bounded intervals to possibly find a new Pareto point and/or improve the already existing Pareto points. For only half of the instances we can find additional Pareto points as seen in Table 5 and the percentage of these newly discovered points is 0.05% on the average. Moreover, we can improve the points only for five out of 20 settings, and the average improvement is 0.03%. These results show that the output of Phase II is a very good approximation to the true Pareto efficient frontier.

We also test the effect of using the improved decomposition algorithm in Phase III of our three-phase algorithm. We use the same setting as in the previous tests except we use the improved decomposition algorithm that includes eligible and effective valid inequalities (i.e., (29)). As we stated previously, since we run the decomposition algorithm over bounded revenue intervals and these intervals are close to each other, we can reuse the already generated cuts as starting point in the decomposition algorithm. We provide the results of our computational study in Table 6, where the first four columns present the results of the improved decomposition algorithm and the last four columns represent the improved decomposition algorithm reusing cuts. In Table 6 "#iter" shows the number of optimization problems solved in Phase III; "%prog" shows the ratio of segments that have been processed to the number of segments after Phase II within the allowed time limit of 1200 seconds for each algorithm.

Table 6 Analysis of Phase III

M	T	α	Imp–DA				Imp–DA–Pool			
			Time (s)	#iter	%prog	#cuts	Time (s)	#iter	%prog	#cuts
10	40	2	0.7	322	100	916	0.5	322	100	701
		5	0.8	374	100	929	0.5	374	100	687
		8	0.7	318	100	928	0.5	318	100	709
		11	0.6	303	100	798	0.4	303	100	616
20	40	2	17.1	1132	100	26201	15.6	1132	100	24489
		5	17.3	1236	100	25758	15.3	1236	100	23942
		8	15.8	1196	100	23334	13.9	1196	100	21462
		11	18.7	1202	100	24699	17.0	1202	100	23011
30	40	2	1196.8	1587	62.8	164166	1189.3	1589	62.9	163494
		5	891.0	1691	71.6	140733	882.8	1691	71.5	140618
		8	772.8	1947	77.7	144380	765.9	1959	78.1	144966
		11	945.2	1915	73.2	155132	931.9	1927	73.6	156356
40	40	2	1200	1773	40.5	131196	1200	1778	40.6	128606
		5	1200	1539	34.5	115731	1200	1542	34.6	113599
		8	1200	1329	42.4	126921	1200	1329	42.4	123308
		11	1200	1850	37.5	126421	1200	1859	37.7	124190
		Avg	542.4	1232	77.5	75515	539.6	1235	77.6	74422

As we see in Table 6, our algorithm provides provably optimal Pareto frontiers for the setting with the market number of 10 and 20, and its progress reduces as the number of markets increases. When we add all previously generated cuts as a pool (implemented via CPLEX user cuts), the average time and the number of cuts slightly decrease, while the number of optimization problems solved and the progress at the end of Phase II increase.

6. Conclusion

In this paper we study an integrated market selection and lot-sizing problem where a pre-defined set of markets with demands throughout a deterministic set of time periods exists. The demand for these markets are satisfied with a production plan that follows a lot-sizing cost structure: fixed cost for each production period and a variable cost for each production unit. The planner makes a decision on which markets to choose to satisfy the demand throughout a planning horizon and obtain certain amount of revenue by this demand satisfaction while planning the production periods. We consider the single-objective problem where the aim is to maximize the total net profit, which is calculated as the difference between the total revenue obtained by market demand satisfaction and the total production cost. We also introduce the multi-objective version of this integrated market selection lot-sizing problem where the aims are to maximize the revenue and minimize the total cost.

We propose a decomposition-based solution algorithm, in which the master problem chooses markets whose demand is to be satisfied. Then, given a market selection, the subproblem for the decomposition algorithm turns out to be a lot-sizing problem for which obtaining the dual variables can be done in polynomial-time. For the multi-objective problem, we propose a three-phase solution approach employing the decomposition algorithm to find the full Pareto frontier. We test our algorithms on a set of instances provided in the literature and show that we substantially improve the solution times.

As a future research direction it is possible to extend the problem so that market selection and production planning is integrated with pricing. Including stochastic demand structure for the markets can be considered as another future research avenue.

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