Tactical Level Planning in Float Glass Manufacturing with Co-Production, Random Yields and Substitutable Products

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We investigate tactical level planning problems in float glass manufacturing. Float glass manufacturing is a process that has some unique properties such as uninterruptible production, random yields, partially controllable co-production compositions, complex relationships in sequencing of products, and substitutable products. Furthermore, changeover times and costs are very high, and production speed depends significantly on the product mix. These characteristics render measurement and management of the production capacity difficult. The motivation for this study is a real life problem faced at Trakya Cam in Turkey. Trakya Cam has multiple geographically separated production facilities. Since transportation of glass is expensive, logistics costs are high. In this paper, we consider multi-site aggregate planning, and color campaign duration and product mix planning. We develop a decision support system based on several mixed-integer linear programming models in which production and transportation decisions are made simultaneously. The system has been fully implemented, and has been in use at Trakya Cam since 2005.

1 Introduction

In this paper, we study tactical level production planning problems in float glass manufacturing. There are some major characteristics of float glass production that are not commonly observed in other production environments:

- The process is continuous, and production cannot be interrupted.
- Yields are random due to random errors scattered on glass surface, resulting from processes that are not fully controllable.
- The process is of co-production type, meaning that several products must be produced simultaneously by the nature of the process.
- Products are substitutable in the sense that demand for a lower quality product can be satisfied by a higher quality product.

The motivation of this study is a real life problem faced at Trakya Cam, Turkey, which is the largest flat glass manufacturer in Turkey. It is also among the top five flat glass manufacturers in Europe and top ten in the world [9]. Float glass is produced from raw materials such as sand, soda ash and limestone, and is used directly in industries such as the construction industry. Furthermore, it is the main input for other flat glass processing industries including automotive glass, processed glass, mirror and laminated glass. We refer the reader to [9] and [22] for a detailed description of Trakya Cam's supply chain.

One of the major issues in Trakya Cam has been assessing the interaction between sales and marketing functions with the production constraints. For example, receipt of an order for a significant amount of Product A eventually results in an increase in production of Product A. However, Product A cannot be produced alone without accompanying Products B and/or C (which can only partially be controlled). Therefore, increase in production of Product A results in an increase in production of Products B and/or C as well. However, since the production increase in Product B and C cannot be adequately foreseen by the sales staff without an accurate production plan, inventory accumulated for Products B and C in an uncontrolled manner are sold by offering promotions, which result in a decrease in profits.

Trakya Cam currently has four facilities (Lüleburgaz, Mersin, Yenişehir, Bulgaria) that can produce float glass at different rates. These facilities are geographically located in relatively distant locations, and transportation costs are significant. Unique characteristics of float glass manufacturing make measurement of "capacity" difficult, and the possible amount of production depends significantly on the product mix. Therefore planning staff cannot define exact capacities of facilities and analyze various trade-offs in capacity planning. Simple rules based on past experience are used for allocating customer demands to facilities. Since transportation costs are not explicitly considered during production planning, logistics costs are high.

In this paper, we describe a mathematical programming based decision support system for solving central planning problems faced in a multi-site float glass manufacturing environment. There are two specific problems we consider:

- *Campaign Duration and Product Mix Planning:* Colored glass is produced in one or two campaigns per year due to high changeover costs. Therefore, campaign planning is important for inventory management of colored glass. The timing of the campaigns, and the production lines used in campaigns are determined by the management. We describe a mixed integer programming model for determining the optimal duration and product mix of campaigns in Section 4.4.
- *Multi-Site Aggregate Planning:* Obtaining a production plan, which is feasible under unique properties of float glass manufacturing, and which minimizes transportation

costs is a difficult problem. Production planners make certain simplifications, and disregard some constraints in order to keep the problem at a manageable size. These simplifications result in infeasibility in the resulting plan, as well as high logistics costs. We describe a multi-objective mixed integer programming model that determines an optimal production plan considering production capabilities, inventory and backlog costs along with transportation costs in Section 4.5.

The system we describe in this paper has been fully implemented and deployed at Trakya Cam. It helps decision making in sales, production and logistics planning. In our models, unique production characteristics of float glass manufacturing are modeled as constraints. Hence, production plans generated are applicable in the production environment. An output of the model related to sales planning is identification of demand that cannot be satisfied by available production capacity. Similarly, production capacity not allocated by existing demand forecast is determined and is used for guiding sales. The system also considers logistics issues by allocating customer demand to production facilities in such a way that transportation costs are minimized. In summary, the main purpose of this study is to develop a mixed integer linear programming based decision support system that unifies sales, planning and logistics decisions.

The rest of the paper is organized as follows: We first provide a review of the related literature in Section 2. We then briefly overview the float glass production, discuss planning issues, and state some simplifying assumptions in Section 3. We develop mixed-integer linear programming models for campaign duration and product mix planning, and multi-site aggregate planning in Section 4. Finally, we compare the new system with the previously employed spreadsheet-based manual planning process in Section 5, and discuss the results that were obtained.

2 Literature Review

Aggregate production planning problems in discrete manufacturing industries have been studied extensively. Nam and Rasaratnam [20] survey several approaches to the problem, and Leachman and Carmon [14] investigate the effect of different mathematical formulations and solution techniques. Basic models assume that demand, capacity requirement and production yields are deterministic, and aim to find an optimal way of utilizing limited capacity to fulfill anticipated demand [3, 4, 19]. An extension of the basic models accounts for multiple objectives such as minimization of late orders and maximization of profitability [7, 10, 20, 24]. On the other hand, planning problems in process industries such as float glass manufacturing has received relatively less attention. Crama et al. [8] discuss the characteristics that separate process industries from discrete manufacturing industries. They also survey several approaches to production planning and flow control in process industries, focusing on the concept of "recipe," which extends the classical Bill of Materials used in discrete manufacturing. Liu and Sherali [17] investigate a coal blending problem, in which the aim is to find an optimal blending plan considering coal specifications, power plant requirements, blending facility capacities and associated costs. They present a mixed integer programming formulation of the problem, and propose a heuristic solution procedure. Pendharkar [23] describe a fuzzy linear program approach to a planning problem in coal mines, and Bitran and Gilbert [5] present an application in semiconductor manufacturing.

Co-production and random yields are commonly observed in process industries. Öner and Bilgiç [21] analyze a variant of the economic lot sizing problem under co-production. Bitran and Leong [6] propose a dynamic programming algorithm for solving a planning problem, and investigate approximate solution techniques. Joly et al. [13] consider a planning problem in petroleum refineries, where several products are produced simultaneously. They model the problem as a mixed-integer nonlinear program, assuming that types and ratios of co-products are known in advance. Bitran and Gilbert [5] describe co-production issues in semiconductor manufacturing. Semiconductor manufacturing involves a highly complex set of processes, some of which cannot be kept under precise control, resulting in co-production of products having several quality levels. On the contrary to [13], the authors of [5] model co-production rates as (uncontrollable) random variables, which follow a probability distribution. Unlike these applications, the co-production structure in float glass manufacturing allows for the type and ratio of products produced simultaneously to be controlled within certain bounds.

Multi-site production planning problems are also well-studied in the literature. Leung et al. [15] present a stochastic optimization approach for the multi-site planning problem faced at a lingerie producer. Levis and Papageorgiou [16] investigate an international pharmaceutical company. Their model accounts for different cost items and tax laws associated with operations in different countries in order to maximize overall profit. For supply chains in which production sites are geographically diverse, transportation costs become significant [12]. In such cases, Guinet [11] shows that making production and transportation decisions simultaneously gives better results than making those decisions sequentially.

There also exist studies directly addressing problems in glass manufacturing. Al-Khayyal et al. [1] consider operational issues in glass manufacturing, and provide a decomposition based solution procedure to determine glass cutting decisions and stacker scheduling. Liu and Tu [18] consider the impact of inventory space limitations to production planning decisions, rather than capacity limitations. The problem mostly applies to glass processing plants, not glass manufacturing plants. Scheduling problem in glass processing plants is also investigated in [2].

3 Problem Definition

In this section we describe the planning problem in float glass manufacturing.

3.1 Float Glass Definitions

- **Float:** Float is a process for producing glass in which raw materials such as sand and soda ash are liquefied, and the solution floats over liquid tin to gain a smooth surface.
- **Coating:** Coating is a chemical process in which one surface of float glass is covered with very thin layers of metal in order to enhance visual and thermal properties.
- **Product groups:** Product groups are identified by color, thickness and coating type of glass.
- **Products:** Products within a product group are identified by size and quality of glass. The three main size groups are jumbo size, machine size and split size. An example product is fume colored, 4 mm, coated, jumbo size, K1 quality glass [22].

3.2 Process Issues in Float Glass Production

Property 1. *Float glass production is an uninterruptible process.*

Float glass production continues 24 hours a day, 365 days a year. It is not possible to stop the glass furnace that melts raw materials to produce liquid glass. Production has to continue throughout the lifetime of the furnace, which is typically over 10 years. Production cannot be interrupted even if process quality is not under control, in which case nonconforming glass is broken and scrapped at the end of the production line.

Property 2. *There exist parallel production lines with different production capabilities.*

Property 3. *Changeover costs associated with color changes are very high.*

Typical color changeover times are 3-7 days. During color changeover, the entire production is nonconforming and is scrapped.

Property 4. *Coated glass can only be produced during daytime.*

Coating is a chemical process that requires daylight, and hence can only be performed during daytime. Since production cannot be stopped, glass with the same color and thickness is produced during nighttime.

Property 5. *Thickness changeover can only be performed gradually.*

Since glass production is a continuous process, thickness changeover has to be performed gradually. In other words, if glass with two different thickness values is to be produced successively, glass with all thickness values in between needs to be produced as well. Furthermore, in order to allow for stabilization of the process, thickness has to be kept constant at intermediate thickness levels for some minimum duration. Thickness changeover times are on the order of hours.

Property 6. *Multiple products have to be produced simultaneously due to random errors scattered on glass surface.*

Float glass production involves complex physical and chemical processes, some of which are not fully controllable. This results in random errors scattered throughout glass surface. There are several types of error, ranging from tiny visual disruptions to serious errors that necessitate scrapping of the glass produced. Since float glass is used in many industries, some errors that are unacceptable for some applications can be disregarded for some other applications. For example, allowances for mirror and automotive glass are much tighter than those for window glass, and glass to be further processed for use in ovens. The diversity in tolerances results in definition of several quality groups based on the number and types of errors allowed.

Glass surface is continuously observed by an optical laser system located before cutting machines. Coordinates and types of nonconformities are determined instantly, and processed by a software system controlling cutting machines. When a nonconformity is detected, cutting policy is changed and glass is collected as lower quality and/or in smaller size. This is how **co-production** is encountered in float glass production: non-controllable errors in process result in simultaneous production of several products. Distribution and density of random errors determine the maximum possible amount of production for each quality and size group.

We construct a hypothetical example in order to clarify the situation. Figure 1a shows a small portion of glass coming from the production line. Glass surface has been scanned by the optical laser system, and an error has been designated by X. Assume that X is out of tolerance limits for glass of quality level K1, but is acceptable for quality level K2. Figure 1b shows the result of cutting the glass in jumbo size. Since one of the pieces contains an error designated by X, it is of quality K2, and the other one is of quality K1. Figure 1c shows the result of cutting the glass in machine size. Since one of the pieces contains an error designated by X, it is of quality K2, and the other three pieces are of quality K1.

Property 7. *Availability of stacking machines limits the number and types of products that can be produced simultaneously.*

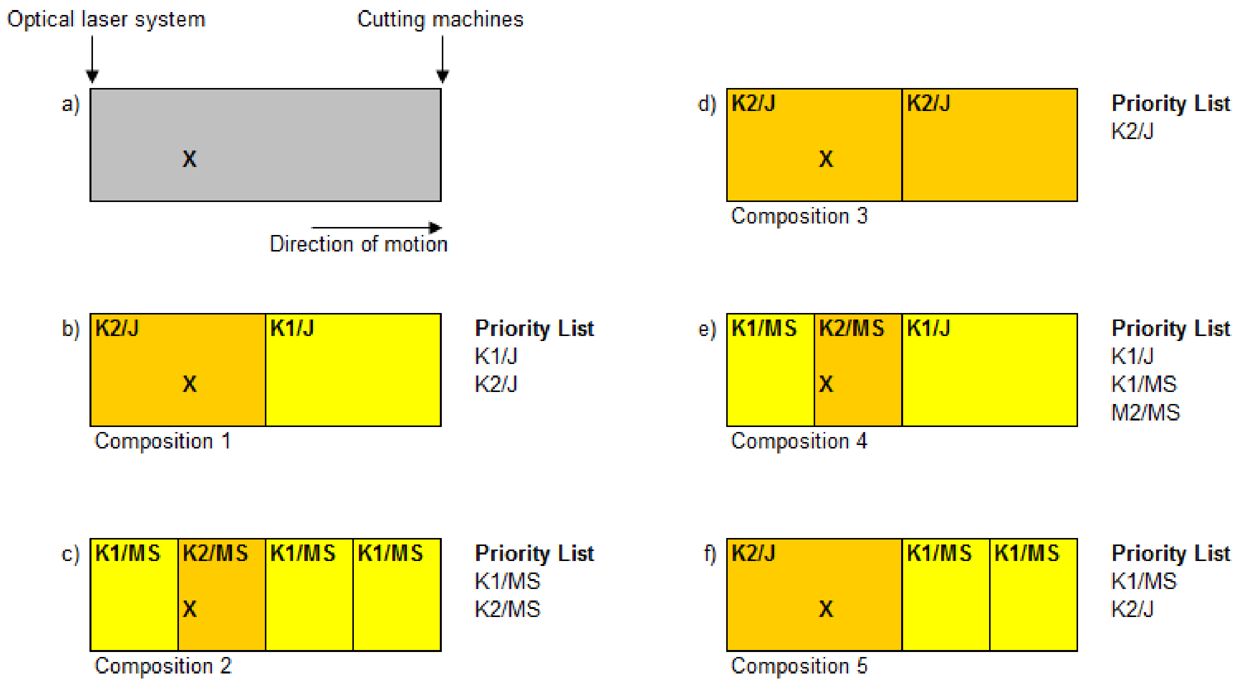

Figure 1: Compositions

After cutting at the end of production line, glass is collected by automated stacking machines and laid on stillages for packaging. Each stacking machine can only be used to collect glass of a specific dimension; it is not possible to collect jumbo size glass on a machine size stacking machine or vice versa. Stacking machines have a cycle time consisting of the time required to collect glass, move it to the required position, lay on stillage, and then return to idle status. Machines cannot be assigned to a new task unless they are idle. Therefore, there is a limit on the availability of each stacking machine based on its cycle time. This is another source of co-production: even if the surface of the glass produced contained no random errors, the fact that stacking capacity for each size group is limited would necessitate simultaneous production of glass within several size groups.

Only one type of product can be collected by a stacking machine at any time. Therefore, the number of products within each size group that can be produced simultaneously is bounded by the number of stacking machines available for that size group. We refer to a cutting policy with respect to the set of products that are produced simultaneously as a **composition**.

3.3 Planning Issues in Float Glass Production

Issue 1. *Production capacity cannot be adjusted through overtime, undertime or subcontracting.*

The fact that stopping production is not an option (Property 1) results in inevitable inventory accumulation for the time periods in which capacity exceeds demand. Furthermore, expanding production capacity by means of overtime or subcontracting is not possible. This results in possible stock-out for time periods in which demand exceeds capacity. Therefore, available capacity can be considered as a "given fact." All production lines have to be fully utilized in a feasible production plan.

Issue 2. *Production capacity depends significantly on the product mix.*

Due to the existence of multiple production lines on which products can be produced at different rates (Property 2), and dynamic co-production possibility (Property 6), capacity cannot be defined independent of the product mix.

Issue 3. *Safety stocks are kept for products.*

Since production plans are based on imperfect forecast information, it is desired to keep safety stocks in order to guard against fluctuations in demand. The safety stock strategy employed is a dynamic one in which it is desired to have inventory level at the end of a period no less than some percentage of the forecasted demand in the next period.

Issue 4. *Products can be substituted for lower quality products within the same size and thickness group.*

From a demand satisfaction point of view, customers demanding glass of a specific size and quality are equally happy with glass having the same size and better quality. Therefore, demand is **substitutable** with respect to quality, but demand substitution is not applicable to products having different dimensions.

In Trakya Cam, demand substitution decisions are made "online" during production. Each available stacking machine is assigned to a specific product. The software system controlling cutting machines is given a **priority list** consisting of a set of products, and the stacking machine each product is assigned to. The software system reads the distribution of errors on glass surface from the optical laser system. Given the error distribution, the system then cuts the glass in such a way that the product with the highest priority that is compatible with the current error distribution is collected by the assigned stacking machine. Usage of priority lists results in implicit product substitution: products that are not in the priority list, and hence are not collected by stacking machines in their actual quality categorization are automatically substituted for products with lower quality that are in the list.

The ideas of product substitution can be demonstrated visually on the hypothetical example given in Figure 1. Given the same error distribution as in Figure 1a, adding only the product K2/J to the priority list and assigning only one stacking machine results in the production output shown in Figure 1d. This policy is equivalent to cutting all glass in jumbo size and substituting K1 glass for K2 glass. In this case, only one end product (K2/J) is produced. In other words, glass of quality K1 is *substituted* for glass of quality K2.

Issue 5. *Cutting policy employed has a significant effect on types and quantities of end products obtained.*

Determining cutting policy is an important planning decision. Although random errors on the glass surface limit possible production of each product (Property 6), the fact that demand is substitutable within the same size group (Issue 4) allows for the application of several cutting policies. Furthermore, it is always possible to cut glass of each size and quality group into several pieces of smaller size having (at least) the same quality group.

Figure 1e shows the result of cutting two machine size pieces instead of the jumbo piece of quality level K2 in Figure 1b. It can be seen that it is possible to cut two machine size glass within quality groups K1 and K2 instead of a single jumbo within size group K2. Similarly, Figure 1f shows the result of cutting two machine size pieces of quality level K1 instead of the jumbo piece of quality level K1 in Figure 1b. The joint effect of surface errors, product substitution and cutting size determination is shown on Figure 1. It can be seen that even for this simple case, where only two size and quality groups are considered, and there exist only a single nonconformity on the glass surface, the total number of cutting policies that can be applied is at least five. Therefore, compositions need to be considered during production planning, and the issue is one of the most interesting issues in float glass manufacturing.

Issue 6. *Colored glass is produced in campaigns.*

As discussed in Property 3, color changeovers have high associated costs, and hence color changeovers need to be planned carefully. Since changeover costs are much more expensive than holding inventory, colored glass is produced in campaigns in order to minimize changeovers. A typical campaign may consist of successive production of clear, light green, dark green, blue and clear glass. Depending on demand patterns, glass of each color is produced in one or two campaigns a year.

Issue 7. *There exist multiple production sites, and transportation costs are significant.*

Most of the float glass demand in Turkey is satisfied by Trakya Cam, and products are exported to several countries worldwide. Therefore, customers are geographically sparse and various transportation methods are used. Production planning department decides on the production plan for each facility, and the facility from which the demand of each customer is to be satisfied. Currently there are four float glass production sites. Transportation of glass is quite expensive, and logistics costs are significant since glass is a fragile material, which can easily be broken during transportation. Specialized pieces of equipment are used in transportation of glass. For example, jumbo size glass is transported by specially manufactured trucks, which are costly to operate.

3.4 Assumptions

In order to build a model that can be solved within a reasonable time, and yet is an adequate representation of the inherent complexity, we make some simplifying assumptions:

Assumption 1. *Changeover time associated with thickness changeover is negligible.*

Although thickness changeover has associated changeover time and cost (Property 5), we assume that thickness changeover time is negligible, and is not modeled explicitly. From the perspective of aggregate production planning, production speed is adjusted to incorporate an average loss due to thickness changeover, based on historical data. This assumption is based on the fact that changeover time for most thickness changes is on the order of hours.

Assumption 2. *There exists a thickness value that corresponds to a significant percentage of demand.*

In the real production environment, it is possible to end a period at any thickness value and begin the next period at the same thickness value. However, sequencing of thickness values is beyond the scope of this paper. Here, we assume that production on each line in each period starts and ends at a specific thickness value, which we refer to as **base thickness value**. This assumption is based on the observation that the ratio of the demand for glass of 4 mm thickness is approximately 60% of the total demand.

Assumption 3. *Glass can only be cut in dimensions on a discrete scale.*

There is a wide range of glass dimensions that can be produced, and product dimensions can be given on a continuous scale. In this paper we assume that only discrete size groups can be demanded by customers, and produced by production lines. Size groups that we consider are jumbo, machine size and split size.

4 Mathematical Model

In this section, we develop a mathematical model for production planning in float glass manufacturing. Table 1 lists the symbols used in our mathematical model, along with their brief descriptions. We first define a **production feasibility set** **(σ)** that determines the set of feasible production plans for a given set of products, which we denote by $\mathcal{P}$, under production capabilities discussed in Section 3.2 and Section 3.3. We later use the production feasibility set as a black-box for campaign duration and product mix planning, and multi-site production planning. The underlying production environment is the same for both planning problems. Therefore, they are similar in terms of production feasibility, but have quite different planning objectives.

Set	Description
C	Set of customers
H	Set of thickness groups
L	Set of production lines
P	Set of products
$\mathcal{P}$	A subset of products used in the production feasibility set
Q	Set of quality groups
S	Set of size groups
T	Set of time periods

Parameter	Description
A_{lt}	Available time on line l at time period t
AI_i	Initial available inventory for product i
b_{ic}	Backlog cost for product i produced for customer c
bs_{ic}	Cost of unsatisfied safety stock for product i produced for customer c
BT	Index of base thickness group
$C(i)$	Coating type of product i (1 if coated, 0 otherwise)
D_{ict}	Demand of customer c for product i at period t
$H(i)$	Index of thickness group for product i
h_{ic}	Inventory holding cost for product i produced for customer c
$L(i)$	Indices of lines that can produce product i
MD_h	Minimum production duration for thickness group h
P_{lt}	Preference of production on line l at time period t
$Q(i)$	Index of quality group for product i
$R(i)$	Index of color for product i
R_{lqs}	Maximum possible ratio of production of quality group q and size group s on line l
$S(i)$	Index of size group for product i
SC_{ls}	Daily stacker capacity for collecting glass of size group s on line l
SP_{il}	Production speed for product i on line l
SS_i	Safety stock level for product i as a ratio of demand in next period
TC_{cls}	Cost of transporting glass of size s from line l to customer c

Variable	Description
B_{ict}	Backlog of product i produced for customer c at the end of period t
BS_{ict}	Unsatisfied safety stock of product i produced for customer c at the end of period t
I_{ict}	Inventory of product i produced for customer c at the end of period t
T_{iclt}	Production duration for product i for customer c on line l at period t
X_{iclt}	Production quantity for product i for customer c on line l at period t
Y_{hlt}	Binary indicator variable for production of products with thickness h on line l at period t

Table 1: Symbols Used in Our Mathematical Model

Before defining our mathematical models we need to explain the reason why production, inventory and backlog variables have a customer index. The reason is that products have several packaging types based on customer requests. Furthermore, customers give exact size specifications within a size group. Therefore in the physical production environment inventory for most customers are kept separately as in our model.

4.1 Mathematical Constraints

Equation (1) relates production duration variables (T) to production quantity variables (X).

$$X_{iclt} = T_{iclt}\ SP_{il} \quad \forall i \in \mathcal{P},\ c \in C,\ l \in L,\ t \in T \tag{1}$$

Equation (2) is the inventory balance constraint that links the production quantity (X), inventory (I), and backlog variables (B) across all time periods.

$$I_{ic(t-1)} - B_{ic(t-1)} + \sum_{l \in L(i)} X_{iclt} - D_{ict} = I_{ict} - B_{ict} \quad \forall i \in \mathcal{P},\ c \in C,\ t \in T,\ t > 0 \tag{2}$$

Equation (3) accounts for satisfaction of safety stock levels, which are represented as a ratio of the demand in the next period (Issue 3). We model desired safety stock levels as "soft constraints," penalizing violations, which are measured by BS-variables.

$$I_{ic(t-1)} \geq D_{ict}\ SS_i - BS_{ic(t-1)} \quad \forall i \in \mathcal{P},\ c \in C,\ t \in T,\ t > 0 \tag{3}$$

Equation (4) forces the value of the indicator variable Y_{hlt} to 1 if production of glass having thickness h is produced in production line l in time period t.

$$\sum_{\substack{i \in \mathcal{P} \\ H(i)=h}} \sum_{c \in C} T_{iclt} \leq A_{lt}\ Y_{hlt} \quad \forall h \in H,\ l \in L, t \in T \tag{4}$$

4.2 Business Model Constraints

Production cannot be interrupted in float glass manufacturing (Property 1). Equation (5) ensures that all production lines are fully utilized in a feasible production plan:

$$\sum_{i \in \mathcal{P}} \sum_{c \in C} T_{iclt} = A_{lt} \quad \forall l \in L,\ t \in T \tag{5}$$

However, in some solution procedures (such as our campaign duration planning algorithm described in Section 4.4), we only consider a subset of the products, and hence, the capacity of the lines need not be utilized completely. In that case, production line capacity constraints need to be of the form:

$$\sum_{i \in \mathcal{P}} \sum_{c \in C} T_{iclt} \leq A_{lt} \quad \forall l \in L,\ t \in T \tag{6}$$

Production feasibility set will be called to have mode STRICT if the capacity constraints are as in Equation (5), and RELAX if Equation (6) applies.

Coated glass can only be produced during daytime (Property 4). In aggregate planning, this means that total production time of coated glass cannot exceed half of the available production time. This requirement can be modeled by the following constraint:

$$\sum_{i \in \mathcal{P}} \sum_{c \in C} C(i)\ T_{iclt} \leq 0.5\ A_{lt} \quad \forall l \in L,\ t \in T \tag{7}$$

Even though this formulation is correct regarding the total production of coated glass, it is inadequate in modeling the coupling of glass produced during daytime and during nighttime: When coated glass is produced during daytime, uncoated glass with the same thickness is produced during nighttime. However, uncoated glass can also be produced during daytime. In aggregate terms, this means that total production of uncoated glass is no less than total production of coated glass for each thickness group:

$$\sum_{\substack{i \in \mathcal{P} \\ H(i)=h}} \sum_{c \in C} (1 - C(i))\ T_{iclt} \geq \sum_{\substack{i \in \mathcal{P} \\ H(i)=h}} \sum_{c \in C} C(i)\ T_{iclt} \quad \forall h \in H,\ l \in L,\ t \in T \tag{8}$$

We note that Equations (6) and (8) imply Equation (7), and hence we do not consider Equation (7) any further.

In order to capture the thickness changeover property described in Property 5, we assume that there exists a base thickness value BT such that production on each line at each period starts at this thickness value (Assumption 2). Based on this assumption, feasible production plans have a special structure regarding thickness groups: If production for any thickness group h is planned, productions for all thickness groups between h and BT have to be planned, too. We enforce this structure as a combination of two constraints:

$$Y_{hlt} \geq Y_{(h+1)lt} \quad \forall h \in H : h \geq BT,\ l \in L,\ t \in T \tag{9}$$

$$Y_{(h+1)lt} \geq Y_{hlt} \quad \forall h \in H : h < BT,\ l \in L,\ t \in T \tag{10}$$

Equation (9) ensures that if production is planned for any thickness group above BT, production should also be planned for the adjacent thickness group with smaller index. This condition is enforced on all thickness groups above BT, and recursively ensures the desired thickness changeover property. Equation (10) handles the symmetric case for thickness groups below BT.

Equation (11) ensures that thickness of glass has to be kept constant at some values for some minimum duration (MD_h) in order to allow for stabilization of the process during thickness changeover (Property 5).

$$\sum_{\substack{i \in \mathcal{P} \\ H(i)=h}} \sum_{c \in C} T_{iclt} \geq MD_h\ Y_{hlt} \quad \forall h \in H,\ l \in L,\ t \in T \tag{11}$$

	K1	K2
J	0.5	1.0
MS	0.75	1.0

Table 2: Bounds on ratio of products in the hypothetical example

Equation (12) limits the production of glass within each size group by the corresponding stacking machine capacity (Property 7).

$$\sum_{\substack{i \in \mathcal{P} \\ S(i)=s}} \sum_{c \in C} X_{iclt} \leq A_{lt}\ SC_{ls} \quad \forall l \in L,\ s \in S,\ t \in T \tag{12}$$

The fact that glass surface contains random errors (Property 6), combined with the possibility of substituting products (Issue 4) results in production in various compositions (Issue 5). Quality groups and size groups are cascaded in the sense that production decision for glass of quality group q and size group s reduces possible production of all quality groups "worse" than q, and all size groups "smaller" than s. Equation (13) is our composition constraint. It states that for each quality group q and size group s, the ratio of production of quality groups $\{1, 2, \ldots, q\}$ and size groups $\{1, 2, \ldots, s\}$ to the total production within a thickness group does not exceed R_{lqs} for each production line. R_{lqs} is a parameter that represents the maximum possible ratio of production of glass within quality group q and size group s on line l. R_{lqs} can be estimated statistically based on historical data.

$$\sum_{\substack{i \in \mathcal{P} \\ H(i)=h \\ Q(i) \leq q \\ S(i) \leq s}} \sum_{c \in C} X_{iclt} \leq \sum_{\substack{i \in \mathcal{P} \\ H(i)=h}} \sum_{c \in C} X_{iclt}\ R_{lqs} \quad \forall h \in H,\ l \in L,\ q \in Q,\ s \in S,\ t \in T \tag{13}$$

In order to clarify the situation, we consider the hypothetical example given in Figure 1. Table 2 shows values of R_{lqs} parameters. It can be seen that ratio of K1/J to total production cannot exceed 0.5 due to random error distribution (Figure 1b). Similarly, the ratio of K1/MS to the total production cannot exceed 0.75 (Figure 1c). In this simple example, the aggregate composition constraints would be:

$$X_{\mathrm{K1J}} \leq 0.5\ [X_{\mathrm{K1J}} + X_{\mathrm{K2J}} + X_{\mathrm{K1MS}} + X_{\mathrm{K2MS}}] \tag{14}$$

$$X_{\mathrm{K1J}} + X_{\mathrm{K1MS}} \leq 0.75\ [X_{\mathrm{K1J}} + X_{\mathrm{K2J}} + X_{\mathrm{K1MS}} + X_{\mathrm{K2MS}}] \tag{15}$$

$$X_{\mathrm{K1J}} + X_{\mathrm{K2J}} \leq 1.0\ [X_{\mathrm{K1J}} + X_{\mathrm{K2J}} + X_{\mathrm{K1MS}} + X_{\mathrm{K2MS}}] \tag{16}$$

$$X_{\mathrm{K1J}} + X_{\mathrm{K2J}} + X_{\mathrm{K1MS}} + X_{\mathrm{K2MS}} \leq 1.0\ [X_{\mathrm{K1J}} + X_{\mathrm{K2J}} + X_{\mathrm{K1MS}} + X_{\mathrm{K2MS}}], \tag{17}$$

where (16) and (17) are redundant for this simple case. We note that all five compositions given in Figure 1 showing various alternative compositions for the hypothetical example are feasible solutions of aggregate composition constraints.

4.3 Production Feasibility Set $\sigma_{\mathcal{P},\text{MODE}}$

Production feasibility set $\sigma_{\mathcal{P},\text{MODE}}$ is a set that defines constraints on decision variables related to a subset of products ($\mathcal{P}$) with respect to a capacity constraint mode. In other words, it defines a polyhedron on the space of decision variables corresponding to the set of feasible production plans for given products. The production feasibility set can be defined as:

$$
\begin{aligned}
\sigma_{\mathcal{P},\text{MODE}} = & \{T_{iclt}, X_{iclt}, I_{ict}, B_{ict}, BS_{ict}, Y_{hlt}\} \\
\text{subject to } & \text{Equations (1)–(4)} \\
& \qquad \text{Equation (5)} \quad \text{if MODE} = \text{STRICT} \\
& \qquad \text{Equation (6)} \quad \text{if MODE} = \text{RELAX} \\
& \qquad \text{Equations (8)–(13)} \\
& \qquad T_{iclt}, X_{iclt} \geq 0 \quad \forall (i \in \mathcal{P}, c, l, t) \\
& \qquad I_{ict}, B_{ict}, BS_{ict} \geq 0 \quad \forall (i \in \mathcal{P}, c, t) \\
& \qquad Y_{hlt} \in \{0, 1\} \quad \forall (h, l, t)
\end{aligned}
$$

4.4 Campaign Duration and Product Mix Planning

Colored glass is produced in campaigns in order to minimize high color changeover time and cost (Issue 6). The main idea behind a campaign is to accumulate enough inventory at the end of the campaign so that the entire demand until the next campaign can be satisfied from inventory. It should be noted that this does not necessarily mean making production for all demand between specified periods; demand can also be satisfied from initial inventory. There are several issues regarding campaign planning:

- *Timing of campaigns:* Since each product has a different initial inventory level and a different demand structure, the time period in which initial inventory is depleted can be different for each product. This is the time before which additional production needs to be planned in order to avoid stock-out for that particular product. In the case of colored glass, the minimum of such time values for all products with the specified color corresponds to the latest time a campaign needs to be planned for that particular color.
- *Sequencing colors within campaigns:* Since changeover times between colors are sequence dependent (Property 3), sequence of colors within campaigns should be determined in such a way that the total changeover time is minimized.

Line	Code	Start Period	Sequence	Color	Changeover
PL1	Campaign1	1	1	Color1	3
	Campaign1		2	Color2	4
	Campaign1		3	Color3	6
PL1	Campaign2	5	1	Color1	3
	Campaign2		2	Color4	5

Table 3: Sample sequence of campaigns

- *Duration of campaigns:* Duration of each campaign must be long enough for accumulating enough inventory to satisfy the demand until the next campaign on the same color. Furthermore, the duration of campaigns need to be minimized in order to avoid accumulating excessive inventory. All other properties of float glass production are also valid for colored glass production. Especially composition (Issue 5), product substitution (Issue 4) and thickness related issues (Property 5) complicate determining the campaign duration.
- *Planning multiple campaigns:* Products with some colors facing rather high demand are produced in more than one campaign per year. For those, timing of the second campaign is an issue. In this case interactions between campaigns need also be planned. If the first campaign satisfies demand of all products until the second campaign, and yet more products need to be produced due to compositions, the co-products should be chosen among those that have demand after the second campaign, if possible. Otherwise, the co-products produced in the first campaign need to be held in inventory for a long time while the second campaign needs to be extended in order to satisfy demand.

Although determining timing of campaigns and the sequence of colors within each campaign automatically is not difficult, Trakya Cam management decided that letting decision makers plan the sequence and timing of campaigns allows them to have more control over the optimization system. On the other hand, calculating optimal duration and product mix of campaigns is a difficult optimization problem, and is tackled by our decision support system. Decision makers determine the start time of campaigns, and colors to be produced within each campaign along with the associated changeover durations in a format similar to Table 3.

There are two color campaigns shown on Table 3. Campaign1 is expected to start at the beginning of period 1 on production line PL1. Color1, Color2 and Color3 will be produced in that order, and associated changeover times are given. After Campaign1, clear glass will be produced until period 5 at the beginning of which Campaign2 will start. Campaign2 consists of production of Color1 and Color4.

In the example given, Color2, Color3 and Color4 are produced only once. Demand of the entire planning period needs to be satisfied in a single production run for those colors. The minimum production duration and product mix for satisfying entire demand under production constraints needs to be found. The situation is more complicated for Color1: it is to be produced twice throughout the year. In the first campaign, entire demand up to period 5 needs to be satisfied at minimum production time, and the rest of the demand needs to be satisfied within the second campaign. However, production within the first campaign that is in excess of demand needs to be planned carefully as well. It is desired that these co-products that are carried in inventory can be used to satisfy demand after period 5 so that inventory costs are minimized. Another benefit of this is a reduction in demand to be satisfied from production within second campaign, resulting in a shorter production duration for the second campaign.

An important observation is that each color is produced independently, and products with different colors have no interaction such as compositions, thickness changeovers except being produced on the same production line on different days. Therefore, products with different colors can be planned in sequential phases by updating available capacities of production lines after each phase.

Model 1. *Campaign duration and product mix planning*

We represent the inventory holding cost of product i for customer c by h_{ic}, and the corresponding backlog cost by b_{ic}. Given these parameters and the definition of the production feasibility set, our mathematical model for planning campaigns for products with color R is given as follows.

$$\text{minimize} \sum_{i\in\mathcal{P},c\in C,t\in T} (h_{ic}\ I_{ict} + b_{ic}\ B_{ict} + bs_{ic}\ BS_{ict}) + \sum_{i\in\mathcal{P},c\in C,l\in L,t\in T} (X_{iclt}\ P_{lt}) \tag{18}$$

$$\text{subject to } \{T_{iclt}, X_{iclt}, I_{ict}, B_{ict}, BS_{ict}, Y_{hlt}\} \in \sigma_{\mathcal{P}=\{p:R(p)=R\},\ \text{RELAX}} \tag{19}$$

Here, P_{lt} are parameters that denote the "preference" of using line l for production at time period t. We initialize P_{lt} parameters as follows: P_{lt} values for time periods in which a campaign is designated to begin are set to 0; P_{lt} values for other time periods are set in an increasing way. In the example given in Table 3 regarding Color1, $P_{21} = P_{25} = 0$, $P_{22} = P_{26} = 1$, $P_{23} = P_{27} = 2 \ldots P_{212} = 11$. P_{lt} values for all other production lines and/or time periods are set to be an arbitrary large value M. In the same example $P_{11} = P_{31} = P_{41} = P_{12} = P_{32} = \cdots = M$. Also note that, RELAX mode is used for the capacity constraints, since we are only considering a subset of the products, and the entire capacity need not be utilized within a period.

Under this setting, production feasibility set guarantees feasibility, and the structure of the objective function guarantees the following properties:

- All demand is satisfied due to the penalty term on backlogs.
- Adequate amounts of safety stocks are kept due to the penalty term on unsatisfied safety stocks.
- Campaign durations are minimized due to the penalty term on inventory carried. Since the feasible region is defined by the the production feasibility set in RELAX mode, the penalty term on the inventory carried guarantees that no inventory that could be eliminated without violating other constraints (such as composition and thickness changeover) is carried in an optimal solution.
- Production is planned as early as possible within each campaign. P_{lt} parameters are initialized in such a way that it is more preferable to produce early within a campaign rather than late. Therefore, production for a later time period is not planned unless production lines are fully utilized in earlier time periods.

Given the campaign planning model (Model 1), which finds an optimal production plan for glass of any color, it can be used within an algorithm for planning all campaigns.

Algorithm 1. *Campaign duration and product mix planning*

Initialize: Sort colors in ascending order of starting period of the first campaign
for all Color R **do**
  for all Production line l, Time period t **do**
    $A_{lt} \leftarrow A_{lt} -$ (changeover time)
    Initialize P_{lt} parameters
  end for
  Apply Model 1 to obtain an optimal production plan for color R
  for all Production line l, Time period t **do**
    $A_{lt} \leftarrow A_{lt} - \sum_{i \in P} \sum_{c \in C} T_{iclt}$
  end for
end for

4.5 Multi-Site Aggregate Planning

The goals of production planning (in the order of priority) can be identified as:

- Obtaining a feasible production plan in terms of production capacity and various interactions between products.
- Satisfying demand as long as production capacity permits, hence minimizing backlogs.
- Minimizing costs incurred by transportation activities.
- Determining production quantity and product mix so that inventory carrying costs are minimized, while appropriate amount of safety stocks are kept.

Clear glass ($R = 0$) is planned after color campaigns by the following mathematical model:

Model 2. *Multi-site aggregate planning*

$$\text{minimize} \quad \sum_{i\in\mathcal{P},c\in C,t\in T} \left[h_{ic}\ I_{ict} + bs_{ic}\ BS_{ict} + b_{ic}\ B_{ict} + \sum_{l\in L}(X_{iclt}\ TC_{clS(i)})\right] \tag{20}$$

$$\text{subject to} \quad \{T_{iclt}, X_{iclt}, I_{ict}, B_{ict}, BS_{ict}, Y_{hlt}\} \in \sigma_{\mathcal{P}=\{p:R(p)=0\},\ \text{STRICT}} \tag{21}$$

Feasible region is defined by $\sigma_{\mathcal{P}=\{p:R(p)=0\},\text{STRICT}}$, and hence the remaining available time after campaigns on all production lines is fully allocated to clear glass. Therefore, if the demand is more than the available capacity, clear glass is made to stock instead of colored glass, parallel to the policy employed by decision makers.

5 Results

In order to examine functionality and correctness of our decision support system, we tested the system using inventory status of products at the beginning of 2005, and monthly sale forecasts for a time period of 12 months. We then compared the production plan generated by our system to the production plan prepared manually by the production planning staff. The staff had been using custom designed spreadsheets on Excel for production planning. Formulas on the spreadsheet were mainly used for error checking purposes, and decision making was completely manual. Since the same data set used for manual planning was used for testing our decision support system, the results are comparable.

As of 2005, Trakya Cam had two plants and four production lines. They had 35 product groups and 140 products. The number of customer groups they targeted was around 30. During the test runs with the planners, we discussed the assignment of cost and penalty parameters used in the objective functions, and tried a number of scenarios to see the impact to the inventory accumulation and transportation needs. Flexibility to modify these parameters and ability to analyze the results easily were found valuable by the planners. Based on the tests, main differences between results obtained are:

Result 1. *Complexity of production planning at individual product level of detail can be handled.*

Production planning was based on color, thickness and coating of glass in manual production planning. Size and quality groups were not considered. The main reason for this is to reduce the dimensional complexity in order to keep the problem at a manageable size for the decision maker. On the other hand, our decision support system is based on products, and handles the additional complexity.

Result 2. *Complexity of production planning at individual customer level of detail can be handled.*

Another simplification made during manual production planning was to group several customers into a few customer groups. For example, even though Trakya Cam exports to more than 20 countries worldwide, all foreign markets were treated as a single customer group. Similarly, all local customers except other Trakya Cam facilities were grouped together. In contrast, individual customers can be modeled in our system, considering their specific requirements and freight costs.

Result 3. *Detailed conditions on composition and demand substitution issues are considered adequately.*

Composition (Issue 5) and product substitution (Issue 4) involve complex relationships between products. Since it is difficult to consider those in manual planning, planning staff could only make the production plan at product group level (Result 1), and did not consider these issues explicitly. Instead, they relied on their experience about feasibility of the plan in terms of compositions. However, comparison of production plans created manually and by our decision support system revealed some infeasibilities in the manual production plan regarding compositions. This was especially the case for some types of colored glass in which demand is quite small compared to clear glass. In one case, more than 70% of colored glass demand was for quality group K1, which is of highest quality level. However, it is not possible to produce K1 quality glass without accompanying glass of lower quality levels due to the composition issue (Issue 5). Investigation of the manual production plan revealed that the amount of planned production was roughly equal to demand for glass of this color, meaning that co-production of lower quality glass was not planned and hence, the composition constraint was violated. The color campaign had to be revised later in the same year in order to correct the mistakes made in planning.

Result 4. *Color campaigns are planned in detail subject to production constraints.*

Colored glass is produced in one or two campaigns each year, and campaign planning is crucial for inventory management of colored glass (Issue 6). Furthermore, since the demand for colored glass is relatively small, composition issues are more relevant in production of colored glass compared to clear glass. Our comparison of the manual production plan with the plan generated by our system showed that:

- Campaign durations for some of the five colors were less than the necessary duration required to produce for demand in subsequent time periods. The main reason for this is that composition issues were disregarded in manual planning (Result 3). Although total amount of colored glass production planned was roughly equal to total demand, it was not possible to satisfy demand of high quality glass without extending campaign duration, and producing some glass of lower quality groups as well.

- On the other hand, durations for some campaigns were more than the sufficient duration. This was especially the case for colors for which more than one campaigns were defined. The main reason is that planners could not optimize compositions in the first campaign so that co-products of the first campaign are required productions for the second campaign. This resulted in excess inventory levels after the first campaign, and extended duration for the second campaign for production of glass that could have been produced in the first campaign.

Result 5. *Conditions on thickness changeovers and minimum production durations are enforced.*

Thickness changeover has to be performed gradually in float glass production (Property 5). However, investigation of the production plan prepared manually revealed some inconsistencies regarding thickness changeovers. For example, for a month, production for thickness groups 4 mm and 6 mm were planned, but no production for the groups 4.2 mm, 5 mm or 5.5 mm were planned. In contrast, our decision support system contains constraints that enforce conditions on thickness changeovers.

Result 6. *Total production quantity and inventory levels are 2% lower than the ones in manual planning.*

Upon comparison of total production quantities, the figure for manual production plan was found to be approximately 2% more than the one for our decision support system. This result suggests that human decision makers tend to choose faster production lines for alternative selection (Property 2) without considering other issues such as transportation costs in detail. However, since the entire demand could already be satisfied, the additional 2% had to be stored in inventory. We note that our decision support system already guarantees that sufficient levels of safety stocks are kept. Therefore, the additional 2% of the production did not have corresponding forecasted demand.

Result 7. *Freight costs are considered explicitly, leading to 10% decrease in transportation costs.*

Since considering transportation costs explicitly is difficult in manual production planning, decision makers were using simple rules based on past experience for assignment of customer demands to facilities. Furthermore, since production planning was based on aggregate customer groups instead of individual customers, it was not possible to consider distance information for individual customers. However, our decision support system considers transportation costs explicitly and customer demands are defined at a more detailed level. We calculated the total transportation cost for the manual production plan, and compared it to the figure for the production plan created by our decision support system. The results indicated a reduction of about 10% in transportation costs without any loss in quality of service.

Result 8. *Planning process speed has improved considerably.*

Previously, it took 3–4 days to prepare a plan (with limited detail, as discussed in the previous results), and hence, a new plan was only generated twice a year to evaluate the validity of the annual production budget. With the decision support system developed in this study, generation of a new plan, including data preparation and reporting, takes at most two hours, which makes it possible to make multiple runs in a month.

These results convinced the managers at Trakya Cam to deploy our system. The decision support system has been in regular use since 2005. Now that running the system and analyzing the results are relatively simple, new tactical plans are generated every 15 days to check the progress of sales and production realizations. In a planning session, planners try various scenarios by changing the weights of various objectives, and analyze the results.

In the mean time, the size of the organization has grown considerably. In 2005, Trakya Cam had two plants and four production lines; now they have four plants and seven lines. Due to the progress of the automotive and construction industries in Turkey, demand for colored glass has increased. So, the number of product groups (now 60), products (now 220) and customer groups (now 40) have also increased. In all this change, the decision support system proved to be invaluable to assess the impact of new investments and new markets.

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