A Mixed Integer Programming Based Solution Methodology for a Scheduling Problem in Tissue Paper Manufacturing

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1 Introduction

In this study, we develop a mixed integer programming (MIP) based solution methodology for the scheduling problem in the largest tissue paper manufacturing company in Turkey. Production process of the company is composed of two major phases: paper production, where tissue paper is produced in bulk quantities, and converting, where large paper rolls are cut into size and packaged.

The manufacturing system consists of four facilities located in different regions of Turkey. Among these facilities, paper is produced only in one plant. After the bulk paper is obtained, it can either be sold directly to customers as bulk tissue paper, or it can be converted to any one of the possible end products, such as bath tissue, paper towels and napkins. The main converting facility is adjacent to the paper production plant. There is a second converting facility owned by the company in a different region in Turkey. The company also works with two contractors with converting facilities in different geographical locations.

The company operates in a very competitive and dynamic market, and needs to respond to changes in the market rapidly without compromising operational integrity. Since the end products are produced to inventory, the company needs to have a proper mix of inventories in the face of changing market dynamics. Profit margins of the sector are relatively limited, and the company must keep its operational costs as low as possible.

Since tissue paper production is performed on large dedicated machines, capacity installed for the paper production phase is considerably larger than the requirements of the converting facilities. Hence, the company needs to balance the possibility of selling the bulk paper as a product and the internal demand generated by the converting facilities.

The scheduling approach presented in this paper is a part of an integrated planning system, which consists of Capacity Planning Module (CPM), Shift Planning Module (SPM) and Scheduling Module (SM). The main objectives of the planning system are to decrease the operational costs such as process, transportation, setup and inventory holding costs; to increase customer service level and to increase responsiveness of the planning department to fast changing market conditions. The planning system works in integration with the Demand Planning, Enterprise Resource Planning (ERP) and Manufacturing Execution System (MES) systems of the company.

Scheduling problem is solved in two phases. In the first phase, a MIP is solved to determine the production batch sizes and machine assignments of the production batches. In the second phase, the production batches are scheduled using heuristic methods.

Optimization based approaches to scheduling has received a significant amount of interest in the literature. We refer the reader to Pochet and Wolsey (2006) for a comprehensive

treatment of production planning using mathematical programming methods. Mendez *et al.* (2006) surveys the use of optimization techniques for solving scheduling problems. Several studies have focused on designing integrated methods for solving production planning and scheduling problems (e.g., Lasserre (1992), Li and Ierapetritou (2010), Maravelias and Sung (2009)). We refer the reader to Belvaux and Wolsey (2001) regarding planning - scheduling models with small buckets to handle setup times explicitly.

2 Two-Phase Scheduling Algorithm

Our scheduling model generates a detailed schedule for the short-term planning horizon, which typically consists of the next two weeks. The generated schedule is based on the net productions requirements (determined by the CPM) and the installed man-hours (determined by the SPM). Generated schedule is released to the MES system.

Our scheduling approach consists of a two-phase scheduling algorithm. In the first phase of scheduling, we solve an optimization model to determine production batch sizes and machine assignments for each batch. The second phase generates a feasible sequence of the resulting batches on the selected machines using a heuristic procedure.

2.1 First Phase: Batch Sizing Model

The mixed-integer programming model that we solve in the first phase is basically an aggregate planning model with additional integer variables to handle batching and machine selection decisions in the face of sequence dependent setups. There are two levels of setups in the converting machines: major setups between different product families, and minor setups between products of the same family. There are also some sequencing rules imposed in the paper production, which arise from the technical restrictions on production of two different paper types after another. Buckets in our models are designed so that, they act as *small buckets* in regards to family setups, while they are *large* in correspondence to minor setups. Each time bucket in our model represents a time span of three days. Hence, our model generates optimum batch sizes and machine assignments for product families, under some simplifying assumptions regarding product sequencing rules and capacity loss due to minor setups. Product sequencing within each family is handled in the second phase of the scheduling process. Main decision variables of the model include basic product based aggregate planning variables (production quantities (x_{ipb}, production quantity for product i in period b through process p; x_{ib}^q, production quantity for finished product i in period b for demand type q), ending inventory levels (I_{ikb}, inventory level of product i in location k at the end of period b; I_{ib}^q, inventory level of product i for demand type q at the end of period b), resource usage amounts (u_{rb}, usage of resource r in period b).) and binary assignment variables that indicate whether a product or a product family is produced in a period on a machine (y_{ipb} indicating whether product i is produced in period b via process p; v_{grb} indicating whether product family g is assigned to resource r in period b).

The constraints of the model include product-based aggregate planning constraints (inventory balance (1, 2, 5), resource capacity (7, 8), minimum lot size (9, 18)), sequencing constraints for product families (11– 17), implementation of priority rules regarding valuation of different sources of demand (such as direct orders from chain markets versus forecasts) (1, 3), and transition rules in paper production (19).

The objective function of our model in the first phase balances the trade-off between minimization of total setup and the prioritized delivery performance. The number of decision variables and constraints are approximately 43.000 and 23.000, respectively. The model looks ahead for one month, that is it takes demand and production decisions made by CPM for a full month into account. However, since we are interested in the detailed

schedule of the next two weeks, we relax the binary variables for the last two weeks. Nevertheless, approximately %30 of all decision variables are binary variables and hence the resulting MIP cannot be solved to optimality within a reasonable amount of time. Therefore, we stop the solution process once an optimality gap defined by the planner is reached. Construction of model and the solution process take approximately five minutes with an optimality gap of %1.

MIP Model:

$$\min \sum_{i \in I} \sum_{p \in P_i} \sum_{b \in B} c_{ip} \, x_{ipb} + \sum_{i \in F} \sum_{q \in Q} \sum_{b \in B} h_i^q \, I_{ib}^q + \sum_{i \in S} \sum_{k \in K_i} \sum_{b \in B} h_{ik} \, I_{ikb}$$

$$+ \sum_{i \in I} \sum_{q \in Q} \sum_{b \in B} u_i^q \, U_{ib}^q + \sum_{i \in S} \sum_{k \in K_i} \sum_{l \in K_i \setminus \{k\}} \sum_{b \in B} f_{ikl} \, z_{iklb} + \sum_{r \in R} \sum_{b \in B} r u_r \, u_{rb}$$

subject to

$$I_{i,b-1}^q + x_{ib}^q - ID_{ib}^q + U_{ib}^q - U_{i,b-1}^q = I_{ib}^q \quad \forall q \in Q, \, \forall i \in F \text{ and } \forall b \in B \tag{1}$$

$$I_{i,0} = \sum_q I_{i,0}^q \quad \forall i \in F \tag{2}$$

$$\sum_{p \in P_i} x_{ipb} = \sum_{q \in Q} x_{ib}^q \quad \forall i \in F \text{ and } \forall b \in B \tag{3}$$

$$\sum_{i \in F} \sum_{p \in P_i} b_{ipj} \, x_{ipb} = DD_{jb} \quad \forall j \in S \text{ and } b \in B \tag{4}$$

$$I_{ik,b-1} + \sum_{p \in P_i(k)} x_{ipb} + \sum_{l \in K \setminus \{k\}} z_{ilkb} - \sum_{l \in K \setminus \{k\}} z_{iklb} - DD_{ib} = I_{ikb} \quad \forall j \in S, \, k \in K_i \text{ and } b \in B \tag{5}$$

$$x_{ipb} \le M \, y_{ipb} \quad \forall i \in F \cup S, \, p \in P_i \text{ and } b \in B \tag{6}$$

$$\sum_{i \in I} \sum_{p \in P_i} a_{ipr} \, x_{ipb} = u_{rb} \quad \forall r \in R \text{ and } b \in B \tag{7}$$

$$u_{rb} \le C_{rb} \quad \forall r \in R \text{ and } b \in B \tag{8}$$

$$x_{ipb} \ge \mu_i \, y_{ipb} \quad \forall i \in F \cup S, \, p \in P_i \text{ and } b \in B \tag{9}$$

$$\sum_{b \in B(t)} x_{ipb} = X_{ipt} \quad \forall i \in F \cup S, \, \forall p \in P_i \text{ and } \forall t \in T \tag{10}$$

$$v_{grb} \ge \chi_{ig} \, y_{ipb} \quad \forall i \in F \cup S, \, \forall p \in P_i(r), \, \forall g \in G \text{ and } \forall b \in B \tag{11}$$

$$v_{grb}^s + v_{grb}^e + v_{grb}^c = v_{grb} \quad \forall g \in G', \, r \in R \text{ and } b \in B \tag{12}$$

$$\sum_{b \in B(t)} v_{grb}^s \le 1 \quad \forall g \in G', \, r \in R \text{ and } t \in T \tag{13}$$

$$\sum_{b \in B(t)} v_{grt}^e \le 1 \quad \forall g \in G', \, r \in R \text{ and } t \in T \tag{14}$$

$$\sum_{b \le b^*} v_{grb}^s \ge v_{grb^*}^e \quad \forall g \in G', \, r \in R \text{ and } \forall b^* \in B \tag{15}$$

$$v_{grb^-} + v_{grb^+} = v_{grb} + 1 \quad \forall g \in G', \, r \in R \text{ and } \forall b, b^+, b^- \in B \text{ such that } b^- < b < b^+ \tag{16}$$

$$x_{ipb^*} = 0 \quad \forall i \text{ such that } \chi_{i,g_j} = 1, \, p \in P_i(r), \, \forall g \in G''', \tag{17}$$

$$\sum_{i \in F} \sum_{p \in P_i(r)} \chi_{ig} \, a_{ipr} \, x_{ipb} \geq v_{grb} \, \tau_g \quad \forall g \in G'', \, r \in R \text{ and } \forall b \in B \tag{18}$$

$$v_{g_j rb} + v_{g_l rb} = v_{g_k rb} + 1 \quad \forall g_j, g_k, g_l \in G^s \text{ such that } g_j \prec g_k \prec g_l \text{ and } \forall b \in B \tag{19}$$

$$x_{ipb} \geq 0, I_{ib}^q, U_{ib}^q, \, x_{ib}^q \geq 0, I_{i0}^q \geq 0, I_{ikb} \geq 0 \tag{20}$$

$$z_{iklb} \geq 0, DD_{ib} \geq 0, u_{rb} \geq 0 \tag{21}$$

$$y_{ipb} \in \{0,1\}, v_{grb} \in \{0,1\}, v_{grb}^s, \, v_{grb}^c, \, v_{grb}^e \in \{0,1\} \tag{22}$$

2.2 Second Phase: Heuristic

The second phase of the scheduling algorithm includes a heuristic, which takes the solution of the optimization model of the first phase as an input, and generates a feasible overall production sequence. To generate the feasible sequence, the algorithm keeps track of the material flow requirements between paper production and converting, and converting and deliveries; and applies a set of product sequencing rules.

The set of sequencing rules that need to be applied are specific to the machine and the product family characteristics of the batches that are being sequenced. Major sequencing issues can be summarized as follows:

1. Sequencing rules regarding product family restrictions;
2. For finished goods: the level of importance of customer orders versus production to forecast;
3. For semi-finished goods: the time that the product becomes critical for the progress of finished good production.

Our heuristic procedure (details of which are omitted for brevity) constructs a continuous-time schedule from the solution of our optimization model, which is based on discrete time buckets. Our heuristic executes very quickly, and does not constitute a bottleneck in the solution procedure.

3 Conclusion

In this study, we provide an overview of the scheduling model that we developed at the largest tissue paper manufacturing company in Turkey. This scheduling model is designed to be a part of integrated planning system with long term capacity planning and shift planning modules. Scheduling model has been in use in the company since March 2011. Observed benefits of using the planning system can be summarized as follows:

- Optimization of inventory flow resulted in an improved inventory mix, hence customer service levels are significantly increased while keeping the total inventory value the same.
- Material and product flow between facilities among the network is also optimized where the scheduling constraints are considered more accurately using mathematical programming. The company reported a 35% decrease in transportation costs within the network.
- Joint modeling of production phases provided a reliable decision support environment in regards to optimum allocation of paper production capacity between external sales opportunities and internal demand.

– The operating environment of the company is highly competitive. It is not uncommon to face each month a very drastic marketing move by one of the competitors. Usage of the planning system improved the responsiveness of the company to take correct position against such perturbations on the estimated state of the market conditions.

– Unlike the planners' off-line spread sheets, the planning system is able to capture the state of shop floor continually and attract planners' attention to the changing conditions which may effect the current schedule. Adapting the current schedule to the new situation or construct a new schedule are the possible activities which can be executed in a short time.

– Integrating the capacity planning with shift planning improved the utilization of resources in the converting plants.

Acknowledgements

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