Optimal Berth Allocation, Time-variant Quay Crane Assignment and Scheduling with Crane Setups in Container Terminals†

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***Abstract*—We focus on the integrated seaside operations in container terminals, namely the integration of berth allocation, quay crane assignment and quay crane scheduling problems. We first develope a mixed-integer linear programming formulation. Then, we propose an efficient cutting plane algorithm based on a decomposition scheme. Our approach deals with berthing positions of the vessels and their assigned number of cranes in each time period in a master problem, and seeks the corresponding optimal crane schedule by solving a subproblem. Our computational study shows that our new formulation and proposed solution method yield optimal solutions for realistic sized instances with up to sixty vessels.**

I. Introduction

As a consequence of the drastic increase in container traffic, the efficient management of container terminals has become a crucial issue and attracted a considerable research effort from various disciplines, including Operations Research [2], [3]. This is a difficult task since there is a myriad of interdependent operations, which can be grouped as the seaside, transfer and yard operations. In this work, we concentrate on the integrated planning of seaside operations, which includes the berth allocation problem (BAP), quay crane assignment problem (CAP) and quay crane scheduling problem (CSP).

Efficient planning of seaside operations has a direct impact on the dwell time of vessels, which is one of the main performance measures at a container terminal. Longer dwell times can have a negative impact on the competitiveness of both the port and companies operating terminals there. This explains the existence of studies in the literature that are concerned with BAP, CAP, CSP, and their integrated versions; namely berth allocation and quay crane assignment problem (BACAP), crane assignment and scheduling problem (CASP) and berth allocation, quay crane assignment and scheduling problem (BACASP). The type of integration we aim for here is the so-called deep integration defined by [4], where subproblems are combined in the form of a single, unified, monolithic mathematical optimization model. A recent example is the work by [5] where the authors make a major assumption: crane assignments to vessels are time-invariant; they are realized when berthing starts, and remain the same until departure. In this work, we extend this line of integration to the more realistic situation where the assignment is time-variant and introduce a formulation that deeply integrates BAP, CAP, and CSP (BACASP), as our first contribution. Our second contribution is a new efficient exact algorithm.

The remainder of the paper is organized as follows. The next section is devoted to the formulation of the new model. In Section 3 we focus on CSP and propose approaches for its efficient solution. Computational results demonstrating the efficiency of the proposed approaches are reported in Section 4. Finally, concluding remarks and future research directions are listed in Section 5.

II. Formulation of the Integrated Model

The new integrated BACASP formulation is mainly based on the following eight assumptions:

1) The planning horizon is divided into time periods of equal length.
2) The berth is continuous and discretized by equal-sized unit berth sections. They are just as large as a single crane can fit in.
3) Each berth section is occupied by no more than one vessel in each time period.
4) The desired berthing sections of the vessels are known. The preference over a berth section may be due to its proximity to the portion of the yard where the containers are to be unloaded (loaded) from (into) the vessel.
5) Each quay crane can work on at most one vessel per time period.
6) Each vessel has a minimum and maximum number of quay cranes that can be assigned to it; and it is long enough to accept the maximum number of cranes.
7) The service of a vessel by the quay cranes starts right after its berthing and lasts without disruption until its departure with possible changes in the number of allocated cranes.
8) The net amount of work q cranes can produce for one period is q^{λ} crane-periods. Here, λ is the interference exponent which can be set to values in $(0, 1]$.

† Full manuscript is currently under revision with the European Journal of Operational Research; an early version appears in [1].

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The parameters and decision variables of our BACASP formulation are listed with their brief descriptions in Table I and Table II, respectively. There are many summations with lower and upper limits of the indices, which are long and difficult to follow in the formulation. We define and organize them in Table III for the sake of notational convenience.

TABLE I. PARAMETERS USED IN OUR MATHEMATICAL MODEL

Par.	Definition
α_{it}	Arrival parameter, which is set to 1 if vessel i can start berthing in period t
$\beta_{itt'}$	Departure parameter, which is set to 1 if vessel i that berths in period t can depart in period t'
δ_i^e	Upper limit on the deviation of vessel i's berthing time from e_i
δ_i^s	Upper limit on the deviation of vessel i's berthing position from s_i
λ	Interference exponent, a real number in $(0, 1]$
θ^m	Setup cost that corresponds to the m^{th} feasible solution of the master problem computed by solving CSSP
ϕ_i^e	Cost of berthing one period later than the expected arrival time for vessel i
ϕ_i^r	Cost of departing one period later than the due time for vessel i
ϕ_i^s	Cost of deviating one unit from the desired berthing section for vessel i
B	Number of berthing sections
e_i	Expected arrival time of vessel i
K	Number of quay cranes
$\overline{k}_i$	Maximum number of cranes that can be assigned to vessel i
$\underline{k}_i$	Minimum number of cranes that can be assigned to vessel i
ℓ_i	Length of vessel i measured in terms of the number of discretized quay sections
$\overline{p}_i$	Upper bound on the processing time of vessel i, it is set to $\left\lceil w_i/\underline{k}_i^{\lambda}\right\rceil$
$\underline{p}_i$	Lower bound on the processing time of vessel i, it is set to $\left\lceil w_i/\overline{k}_i^{\lambda}\right\rceil$
r_i	Departure due time of vessel i
s_i	Desired berthing section of vessel i
T	Number of time periods
V	Number of vessels
w_i	Workload of vessel i in crane-periods

TABLE II. DECISION VARIABLES USED IN OUR MATHEMATICAL MODEL

Var.	Definition
θ	Variable lower bound on the total setup cost
a_t	1 if there is a berthing vessel in period t, 0 otherwise
d_t	1 if there is a departing vessel in period t, 0 otherwise
$x_{ijtt'}$	1 if vessel i berths at section j in period t and departs in period t', zero otherwise
y_{it}	1 if vessel i is in the berth in period t, 0 otherwise
z_{iqt}	1 if the number of cranes assigned to vessel i in period t is q, zero otherwise

Before we explain the objective function and the constraints of our model, we would like to introduce several quantities that are used as lower and upper limits in the summations. Their usage helps to reduce the number of terms involved. The first two of these are $(s_i - \delta_i^s)$ and $(s_i + \delta_i^s)$. They appear in the summation limits for berth section index j in (1)–(3), provide lower and upper bounds on the berthing position s_i of vessel i, respectively. Here, δ_i^s can be obtained by generating a feasible solution of BACASP with an objective value Z_f. Since each unit deviation of vessel i's berthing position from the desired one contributes ϕ_i^s units to the objective value, δ_i^s can be set equal to $\lceil Z_f/\phi_i^s \rceil$. In a similar fashion, we can set an upper limit on the tardiness of vessel i with respect to its expected arrival time e_i. It is denoted by δ_i^e and can be set to $\lceil Z_f/\phi_i^e \rceil$, where ϕ_i^e is the cost of vessel i's berthing one period later than e_i.

TABLE III. LOWER AND UPPER LIMITS OF THE INDICES

Par.	Explanation	Original term
σ_i^1	The smallest index of the berth section for vessel i	$\max(1, s_i - \delta_i^s)$
σ_i^2	The largest index of the berth section for vessel i	$\min(B - \ell_i + 1, s_i + \delta_i^s)$
$\eta_i^1(t)$	The earliest period in which vessel i can depart.Here t is the berthing time of the vessel	$t + \underline{p}_i - 1$
$\eta_i^2(t)$	The latest period in which vessel i can depart. Here t is the berthing time of the vessel	$\min(t + \overline{p}_i - 1, T)$
τ_i	The latest period in which vessel i can berth	$\min\left(T - \underline{p}_i + 1, e_i + \delta_i^e\right)$

We start to explain our formulation with the objective function (1).

$$\min \sum_{i=1}^{V} \sum_{j=\sigma_i^1}^{\sigma_i^2} \sum_{t=e_i}^{\tau_i} \sum_{t'=\eta_i^1(t)}^{\eta_i^2(t)} \{\phi_i^s |j - s_i| + \phi_i^e (t - e_i) + \phi_i^r \max(0, t' - r_i)\} x_{ijtt'} + \theta. \quad (1)$$

It consists of the minimization of the total cost and is basically the sum of two terms. The first one is obtained by summing up over the vessels the costs of deviation from the desired berthing section s_i, the costs of berthing later than the expected arrival time e_i (i.e. late arrival), and the costs of departing later than the departure due time r_i that marks the time at which the loading/unloading operation of the vessel should be finished (i.e. late departure). These three cost components are formulated by means of the binary variable $x_{ijtt'}$, which represents the allocation of berths to vessels for a time interval; it is set to one if vessel i berths at section j from time t until its departure time t', and zero otherwise. The second term (i.e., the variable θ) is a lower bound on the total setup cost $Q(\mathbf{x}, \mathbf{z})$ associated with crane relocations when feasible $\mathbf{x}$ and $\mathbf{z}$ values are given with respect to constraints (2)–(12). It is obtained by solving the subproblems created via a Benders-like decomposition scheme [6], as will be explained later in Section 4.

We use binary variables $x_{ijtt'}$ and adopt the position assignment approach in formulating BAP constraints (2) and (3) given below.

$$\sum_{j=\sigma_i^1}^{\sigma_i^2} \sum_{t=e_i}^{\tau_i} \sum_{t'=\eta_i^1(t)}^{\eta_i^2(t)} x_{ijtt'} = 1 \quad i = 1, \ldots, V, \tag{2}$$

$$\sum_{i=1}^{V} \sum_{j=\max\left(1, \hat{j}-\ell_i+1\right)}^{\min(B-\ell_i+1, \hat{j})} \sum_{t=e_i}^{\min\left(\tau_i, \hat{t}\right)} \sum_{t'=\max\left(\eta_i^1(t), \hat{t}\right)}^{\eta_i^2(t)} x_{ijtt'} \leq 1 \qquad \hat{j} = 1, \ldots, B; \hat{t} = 1, \ldots, T. \tag{3}$$

The next three sets of constraints are related to CAP. Let binary decision variable z_{iqt} equal one if the number of cranes assigned to vessel i in period t is q, zero otherwise. Constraints (4) determine the number of cranes assigned to a vessel in a period, while (5) guarantee that the assignments satisfy the required workload of the vessels (measured by crane-periods). Constraints (6) ensure that the number of assigned cranes does not exceed the number of available ones. Notice that in (5) we use the interference exponent λ in order to model the productivity loss due to the interference between the cranes.The productivity obtained by assigning q cranes to vessel i in time period t is q^λ, which is less than q.

$$\sum_{q=0}^{\overline{k}_i} z_{iqt} = 1 \qquad i = 1, \ldots, V; \; t = e_i, \ldots, T, \tag{4}$$

$$\sum_{t=e_i}^{T} \sum_{q=0}^{\overline{k}_i} q^\lambda z_{iqt} \geq w_i \qquad i = 1, \ldots, V, \tag{5}$$

$$\sum_{\substack{i=1 \\ e_i \leq t}}^{V} \sum_{q=0}^{\overline{k}_i} q z_{iqt} \leq K \qquad t = 1, \ldots, T. \tag{6}$$

BAP and CAP constraints given above are independent of each other. To couple them we define a new binary decision variable y_{it}. It is an auxiliary variable and its value equals one if vessel i is berthed in period t, zero otherwise. Constraints (7) relate variables y_{it} with variables $x_{ijtt'}$. Lower and upper bound constraints (8) on the number of cranes used for vessel i in period t form the relationship between y_{it} and z_{iqt}.

$$y_{i\hat{t}} = \sum_{j=\sigma_i^1}^{\sigma_i^2} \sum_{t=e_i}^{\min\left(\tau_i, \hat{t}\right)} \sum_{t'=\max\left(\eta_i^1(t), \hat{t}\right)}^{\eta_i^2(t)} x_{ijtt'} \quad i = 1, \ldots, V; \quad \hat{t} = e_i, \ldots, T, \tag{7}$$

$$\underline{k}_i y_{it} \leq \sum_{q=0}^{\overline{k}_i} q z_{iqt} \leq \overline{k}_i y_{it} \qquad i = 1, \ldots, V; \quad t = e_i, \ldots, T. \tag{8}$$

We formulate CSP constraints by means of new binary decision variables a_t and d_t. a_t equals one if there is an arrival to the berth in period t, and zero otherwise. Similarly, d_t equals one if there is a departure from the berth in period t, and zero otherwise. Constraints (9) identify whether or not there is an arrival in period t. They are followed by constraints (10), which are their departure-related versions.

$$a_t \leq \sum_{i=1}^{V} \sum_{j=\sigma_i^1}^{\sigma_i^2} \sum_{t'=\eta_i^1(t)}^{\eta_i^2(t)} \alpha_{it} x_{ijtt'} \leq V a_t \quad t = 1, \ldots, T, \tag{9}$$

$$d_{t'} \leq \sum_{i=1}^{V} \sum_{j=\sigma_i^1}^{\sigma_i^2} \sum_{t=e_i}^{\tau_i} \beta_{itt'} x_{ijtt'} \leq V d_{t'} \quad t' = 1, \ldots, T, \tag{10}$$

Notice the use of the arrival and departure coefficients α_{it} and $\beta_{itt'}$, which determine whether or not vessel i can begin or end berthing in period t. Their values are set as:

$$\alpha_{it} = \begin{cases} 1 & \text{if } e_i \leq t \leq \tau_i \\ 0 & \text{otherwise} \end{cases}$$

and

$$\beta_{itt'} = \begin{cases} 1 & \text{if } \eta_i^1(t) \leq t \leq \eta_i^2(t) \\ 0 & \text{otherwise} \end{cases} .$$

The following two sets of constraints relate vessel arrivals and departures with the number of assigned quay cranes: the number of cranes assigned to vessel i can change from period $t-1$ to period t only if arrival of another vessel occurs in period t or departure of another vessel occurs in period $t-1$.

$$\sum_{q=0}^{\overline{k}_i} q z_{iqt} - \sum_{q=0}^{\overline{k}_i} q z_{iq(t-1)} \leq \overline{k}_i \left(a_t + d_{t-1}\right) \quad i = 1, \ldots, V; \quad e_i + 1 \leq t \leq T, \tag{11}$$

$$\sum_{q=0}^{\overline{k}_i} q z_{iq(t-1)} - \sum_{q=0}^{\overline{k}_i} q z_{iqt} \leq \overline{k}_i \left(a_t + d_{t-1}\right) \quad i = 1, \ldots, V; \quad e_i + 1 \leq t \leq T. \tag{12}$$

The last set of constraints determines the total crane setup cost by setting the value of variable θ to the smallest possible lower bound on it. Since the decision variables in constraints (2)–(12) are binary, there is only a finite number of values for variables $x_{ijtt'}$ and z_{iqt} that satisfy these constraints. Let $m = 1, \ldots, M$ index these feasible solutions. Consider the following set of inequalities:

$$\theta \geq \theta^m - \theta^m \times \left[\left(V - \sum_{i=1}^{V} \sum_{\{j,t,t': x^m_{ijtt'}=1\}} x_{ijtt'} \right) + \sum_{i=1}^{V} \left((T - e_i + 1) - \sum_{t=e_i}^{T} \sum_{\{q: z^m_{iqt}=1\}} z_{iqt} \right) \right] \quad m = 1, \ldots, M \tag{13}$$

where $\theta^m = Q(\mathbf{x}^m, \mathbf{z}^m)$, denotes the setup cost obtained by solving the crane scheduling subproblem (CSSP) for given $\mathbf{x}^m$ and $\mathbf{z}^m$. In other words, θ^m represents the crane setup cost corresponding to the m^{th} feasible solution of the problem.

In principle it is possible to generate all constraints of type (13) since M is a finite number. Then one can solve the formulation (2)–(13) to find an optimal solution of BACASP. Since the objective function aims to minimize θ, constraints (13) ensure that θ equals the setup cost corresponding to an optimal solution of the subproblem. However, this approach is not applicable in practice since the number of inequalities (13) can be very large. In order to solve the above model efficiently we propose the cutting plane procedure given in Algorithm 1.

Algorithm 1

Initialization $UB \leftarrow \infty, LB \leftarrow 0, m \leftarrow 1$
Initialize the master problem with constraints (2)–(12)
loop
  Solve the master problem to optimality
  Record the optimal objective function value as the current lower bound LB; optimal $\mathbf{x}, \mathbf{z}$ as $\mathbf{x}^m, \mathbf{z}^m$
  Calculate the setup cost θ^m corresponding to $\mathbf{x}^m, \mathbf{z}^m$ by solving CSSP.
  Compute objective value obj^m (1) corresponding to $\mathbf{x}^m, \mathbf{z}^m, \theta^m$
  if $obj^m \leq UB$ **then**
    $UB \leftarrow obj^m, \mathbf{x}^* \leftarrow \mathbf{x}^m, \mathbf{z}^* \leftarrow \mathbf{z}^m, \theta^* \leftarrow \theta^m$
  end if
  if $LB = UB$ **then**
    Stop; $\mathbf{x}^*, \mathbf{z}^*, \theta^*$ are optimal
  else
    Add constraint (13) corresponding to $\mathbf{x}^m, \mathbf{z}^m, \theta^m$ to the master problem
    $m \leftarrow m + 1$
  end if
end loop

We initially relax all constraints (13) and solve the relaxed master problem. The optimal objective function value is a lower bound (LB) on the optimal value of BACASP. We calculate the optimal setup cost corresponding to the current solution $(\mathbf{x}^m, \mathbf{z}^m)$ by solving the corresponding CSSP. Quantity obj^m corresponds to the objective value of the feasible solution $\mathbf{x}^m, \mathbf{z}^m, \theta^m$ of BACASP and thus forms an upper bound (UB) on its optimal value. We update UB if we obtain a better upper bound. If LB becomes equal to UB, we terminate the algorithm as the current solution is optimal. Otherwise, we add the constraint (13) corresponding to $\mathbf{x}^m, \mathbf{z}^m$ to the master problem and solve it again. The use of optimality cuts such as (13) is a known solution strategy for intractable mathematical programming problems ever since Benders' seminal work [6].

III. Determination of the Total Setup Cost θ^m

The subproblem considered in this section, which we refer to as the *Crane Scheduling Subproblem* (CSSP), focuses on assigning quay cranes to optimal work positions in each period given the berth allocations of the vessels and number of cranes that will serve them (i.e., a feasible solution of BACAP) with the objective of minimizing the total setup cost due to crane relocations on the berth over the planning horizon. We discuss two different solution approaches each of which solves a different network optimization problem. Both formulations make the following assumptions:

1) A setup cost incurs if one of the following three events occurs: a crane starts working on a new vessel when it was serving another one, a crane is assigned to a vessel when it was idle, and a crane stops serving a vessel and becomes idle.
2) Cranes can serve any berthed vessel but are restricted to move along a single line, and hence cannot pass each other.
3) Cranes are initially labeled according to their order along the berth starting from the beginning.
4) In any period, idle cranes wait for their new assignments at the available positions with given capacities located before and after the vessels.
5) Cranes are identical; the differentiation in the setup costs is due to the variance in the working conditions.

A. *Minimum Cost Flow Formulation and Branch-and-bound*

The minimum cost flow problem (MCFP) formulation of CSSP is based on a directed, layered, single source and single sink network. The only node of the first layer is the source node with supply equal to the total number of quay cranes. Similarly, the last layer consists of a single node as well; it is the sink node with demand equal to the total number of quay cranes. The remaining ones belong to internal layers and are pure transshipment nodes.

Each vessel berthed in layer l is represented by a node, whose demand is equal to the number of assigned cranes. These nodes are called vessel nodes and they are ordered starting from the bottom of the layer to the top in accordance with their position in the berth from the beginning to the end. There is a second type of node below and above each vessel node. They are called wait nodes and represent the waiting area for idle cranes. In any layer, the number of wait nodes is one larger than the number of vessel nodes. To summarize, by letting n_l denote the number of berthed vessels in time interval l, there are n_l vessel nodes and $n_l + 1$ wait nodes. Hence, the total number of nodes in layer l is $2n_l + 1$ and nodes with an even index correspond to a vessel node, while those with an odd index represent wait nodes.

Each vessel node in layer l has two copies. Therefore, we name the first one as the original vessel node and the duplicate as the copy vessel node. The original node i_l and its copy i'_l, for even i_l are connected by arc (i_l, i'_l). Since the demand of an original vessel node in layer l is equal to the number of assigned cranes g_{il}, so is the demand of a copy vessel node. Therefore, both the lower bound $\underline{u}_{i_l i'_l}$ and upper bound $\overline{u}_{i_l i'_l}$ on the capacity of the arc (i_l, i'_l) are set to g_{il}, which implies that the flow on the arc (i_l, i'_l) for even i_l is equal to g_{il}. Similar to the vessel nodes, wait nodes also have duplicates. An original wait node i_l and its copy i'_l for odd i_l are connected by an arc whose capacity has a lower bound $\underline{u}_{i_l i'_l} = 0$ and an upper bound $\overline{u}_{i_l i'_l} = h_{i_l}$, which is the capacity of the waiting area i_l when i_l is odd.

Let $\mathcal{V}$, $\mathcal{V}'$, $\mathcal{W}$, and $\mathcal{W}'$ denote the set of original vessel nodes, copy vessel nodes, original wait nodes, and copy wait nodes, respectively. Then, the overall network $\mathcal{N}$ has $(\mathcal{V} \cup \mathcal{V}' \cup \mathcal{W} \cup \mathcal{W}' \cup \{s_1, s_2\})$ nodes, where s_1 and s_2 are the source and sink nodes, respectively. By generating the duplicates of the vessel and wait nodes, all the nodes become pure transshipment nodes except s_1 and s_2. The supply of s_1 and the demand of s_2 are equal to K, that is the total number of cranes. There is an outgoing arc connecting s_1 and an original node of the first layer, and an incoming arc connecting a copy node of layer L to s_2. The capacity of these arcs, as well the one of arcs (i'_l, j_{l+1}), which connect a copy node i'_l in layer l with an original node j_{l+1} in layer $l+1$, has zero lower bound and an upper bound of K.

As can be noticed, the flows on the arcs of this network correspond to crane relocations or movements from waiting areas to vessels, from vessels to vessels (this includes the case where a crane continues serving the same vessel or starts serving a new vessel at the same berth section), and from vessels to waiting areas in each time interval. The costs associated with these relocations are defined as unit flow costs. The unit flow cost on arc (i'_l, j_{l+1}) between two layers, where $1 \leq i'_l \leq 2n_l+1$ and $1 \leq j_{l+1} \leq 2n_{l+1}+1$ is equal to $c_{i'_l j_{l+1}}$. Similarly, the unit flow cost of the arcs between the source node s_1 and the original nodes in the first layer is given as $c_{s_1 i_1}$, while that of the arcs between the copy nodes in layer L and the sink node s_2 is equal to $c_{i_L s_2}$. The arcs (i_l, i'_l) in each layer $l = 1, \ldots, L$ have a zero unit flow cost, because these arcs are only used for treating vessel and wait nodes as transshipment nodes.

The subproblem becomes an ordinary MCFP on the described layered network if crane crossing is allowed, and it can be solved very efficiently. The objective function

$$\theta^m = \sum_{i_1=1}^{2n_1+1} c_{s_1 i_1} f_{s_1 i_1} + \sum_{l=2}^{L-1} \sum_{i'_{l-1}=1}^{2n_{l-1}+1} c_{i'_{l-1} i_l} f_{i'_{l-1} i_l} + \sum_{l=2}^{L-1} \sum_{i_{l+1}=1}^{2n_{l+1}+1} c_{i'_l i_{l+1}} f_{i'_l i_{l+1}} + \sum_{i_L=1}^{2n_L+1} c_{i_L s_2} f_{i_L s_2}, \tag{14}$$

which represents the total cost of flows in the network, is essentially the total setup cost due to the crane relocations. It is minimized subject to the flow balance equations

$$\sum_{i_1=1}^{2n_1+1} f_{s_1 i_1} = K \tag{15}$$

$$f_{i_l i'_l} - \sum_{i'_{l-1}=1}^{2n_{l-1}+1} f_{i'_{l-1} i_l} = 0 \qquad i_l = 1, \ldots, 2n_l+1; \quad l = 2, \ldots, L-1 \tag{16}$$

$$\sum_{i_{l+1}=1}^{2n_{l+1}+1} f_{i'_l i_{l+1}} - f_{i_l i'_l} = 0 \qquad i'_l = 1, \ldots, 2n_l+1; \quad l = 2, \ldots, L-1 \tag{17}$$

$$-\sum_{i'_L=1}^{2n_L+1} f_{i'_L s_2} = -K, \tag{18}$$

and lower and upper bounds on the flow variables

$$0 \leq f_{s_1 i_1} \leq K \qquad i_1 = 1, \ldots, 2n_1+1 \tag{19}$$

$$\underline{u}_{i_l i'_l} \leq f_{i_l i'_l} \leq \overline{u}_{i_l i'_l} \qquad i_l = i'_l = 1, 2, \ldots, 2n_l+1; \quad l = 2, \ldots, L-1 \tag{20}$$

$$0 \leq f_{i'_L s_2} \leq K \qquad i'_L = 1, \ldots, 2n_L+1, \tag{21}$$

which are set to $\underline{u}_{i_l i'_l} = \overline{u}_{i_l i'_l} = g_{il}$ for $i_l = i'_l = 2, 4, \ldots, 2n_l$, $\underline{u}_{i_l i'_l} = 0$ and $\overline{u}_{i_l i'_l} = h_{i_l}$ for $i_l = i'_l = 1, 3, \ldots, 2n_l+1$, as mentioned earlier. The determination of θ^m for the new optimality cut consists of solving the MCFP when crossing is not allowed.

Branch-and-bound algorithm essentially corrects the crossings between the paths of the cranes that occur in an optimal solution of the MCFP. Obviously, the paths of two cranes can cross more than once. Since path crossings imply arc crossings between two consecutive layers of the network on the path of two cranes, it is possible to focus on a single crossing each time there is need for branching at a node of the branch-and-bound tree. A summary of the results on minimum cost noncrossing flow problem can be found in a recent work [7].

TABLE IV. PERFORMANCE OF THE NEW METHOD: REALISTIC INSTANCES

V	Master Problem	CSSP with BB	CSSP with SP	Overall Problem with BB	Overall Problem with SP	Tree size
3	2.7	0.01	0.02	10.7	10.8	6
6	35.1	0.01	0.02	1,293.1	1,293.8	13
9	77.9	0.02	0.03	543.5	544.1	19
12	95.2	0.02	0.04	2,951.7	2,954.3	25
15	99.6	0.03	0.04	4,482.3	4,484.8	29
18	119.6	0.03	0.05	4,543.2	4,545.1	32
21	159.2	0.04	0.06	10,493.6	10,496.8	35
Avg.	84.19	0.024	0.037	3,474.01	3,475.67	22.71

B. Dynamic Programming and Shortest Path Formulation

To formulate the CSSP as a shortest path problem we construct the layered network $\overline{\mathcal{N}} = (\overline{\mathcal{V}} \cup \{\overline{s}_1, \overline{s}_2\}, \overline{\mathcal{A}})$. The layers represent again the L intervals during which there is no change in the berthed vessels. Each node in $\overline{\mathcal{V}}$ in layer l is for a crane-to-vessel assignment combination and arcs in $\overline{\mathcal{A}}$ are for the relocations between the layers. The nodes $\overline{s}_1$ and $\overline{s}_2$ are, respectively, the source and sink nodes. There is an arc connecting $\overline{s}_1$ to the nodes of layer 1 and the nodes of layer L to $\overline{s}_2$. Each combination of an interval represents a sequence of the cranes along the berth after they are allocated to the vessels subject to the non-crossing constraints. Essentially, the network $\overline{\mathcal{N}}$ is a representation of the states of Park and Kim's dynamic programming (DP) formulation [8]. The recursion given in Park and Kim's formulation can be restated as

$$TSC(l) = \min_{\substack{i=1,2,\ldots,S_l;\\ j=1,2,\ldots,S_{l+1}}} \{SC(i,j) + TSC(l+1)\} \qquad l = 0, 1, \ldots, L. \tag{22}$$

Here, S_l and S_{l+1} are the number of non-crossing sequences at layers l and $l+1$, $TSC(l)$ is the minimum total setup cost for periods $l, l+1, \ldots, L$ and $SC(i,j)$ is the cost of the setups required to obtain crane sequence j from sequence i, which can be calculated by summing up the individual costs associated with the setups transforming sequence i to sequence j. Clearly, layers 0 and $L+1$ have only single nodes, namely $\overline{s}_1$ and $\overline{s}_2$ representing initial and terminal crane sequences. Hence, $TSC(L+1) = 0$ and $TSC(0) = \theta^m$.

IV. COMPUTATIONAL STUDY

In this section we perform computational tests in order to assess the performance of the new model and solution methods. They can be grouped in two major sets. The first one is realized with a subset of the test instances given in [9] and belong to a real container terminal. We select seven of the instances with $V = 3, 6, 9, 12, 15, 18, 21$ vessels. The second set consists of larger test problems. They are generated randomly.

TABLE V. AVERAGE NUMBER OF CUTS PER GROUP OF TEST INSTANCES

V	Num. Iters.	Num. Cuts
20	73	70
25	96	92
30	138	133
35	173	168
40	199	194
45	230	224
50	259	252
55	295	290
60	341	335

The cost coefficients associated with berthing away from the desired berth section, late berthing, and late departure are selected as $\phi_i^s = 1000$, $\phi_i^e = 1000$, and $\phi_i^r = 2000$. We assume that the setup costs related to crane relocations are all equal and have the value $\phi^f = 50$. In all instances there are $K = 12$ cranes, $B = 24$ berth sections each having a length of 50 meters, and $T = 200$ periods each corresponding to an hour. The interference exponent λ is set to 0.95. The experiments are carried out on a computer with Intel Xeon 3.16 GHz processor and 32 GB of RAM working under Windows 2003 Server operating system. We use commercial solver CPLEX 12.6 to solve linear and integer programming problems.

Table IV shows the average CPU time in seconds per optimality cut to solve the main model, the CPU time to solve CSSP by the branch-and-bound (BB) algorithm and the shortest path algorithm (SP), and the size of the BB tree averaged over the number of cuts added to the master problem. Tree size statistics is only for BB and SP are the same.

In order to test the performance of the new approach better we have generated larger ones randomly. We vary the number of vessels V from 20 to 60. T is 400, 500, and 600 when V takes on values between 20 and 40, 41 and 50, and 51 and 60, respectively. The length of the berth, the number of cranes available and the cost coefficients are the same as before. The vessel-dependent parameters length (ℓ_i), desired berth section (s_i), workload (w_i), lower bound ($\underline{k}_i$) and difference between the bounds ($\overline{k}_i - \underline{k}_i$) on the crane numbers are generated from uniform distributions respectively between 3 and 8 berth sections, 1 and $B - \ell_i + 1$, 10 and 120, 1 and 5, and 0 and 5. Notice that $B = 24$ 50-meter-sections, which makes a 1200 meters long berth. The vessel lengths are generated between 3 and 8 berth sections, which means between 150 and 400 meters. For each number of vessels we generate five instances. Table V shows the average number of iterations and cuts generated by Algorithm 1 for each group.

Table VI reports the average CPU times spent for solving the master problem and the overall problem over five instances, for each group. We can observe that very large problem instances with $T = 600$ and $V = 60$ can be solved to optimality. We also observe that BB is slightly faster than SP in solving CSSP, which causes a slight decrease in the overall problem's solution time.

TABLE VI. PERFORMANCE OF THE NEW METHOD: LARGE INSTANCES

V	Master Prob.	CSSP with BB	CSSP with SP	Overall Problem with BB	Overall Problem with SP	Tree Size
20	124.5	0.04	0.05	8,585.2	8,587.3	42
25	163.2	0.04	0.06	14,845.7	14,847.9	55
30	218.0	0.05	0.06	28,768.2	28,771.1	57
35	226.9	0.06	0.07	37,641.3	37,644.2	66
40	248.3	0.06	0.07	47,893.9	47,897.2	69
45	298.2	0.07	0.08	66,192.6	66,196.4	75
50	349.4	0.07	0.08	87,690.7	87,695.1	79
55	374.5	0.07	0.09	108,223.3	108,228.3	86
60	420.8	0.08	0.09	140,505.5	140,511.2	89
Avg.	346.26	0.089	0.093	77,178.49	77,196.95	88.29

V. CONCLUSIONS

Our computational study demonstrates that the new formulation coupled with the proposed solution method gives optimal solutions for realistic size problem instances. Also it turns out that the MCFP relaxations based branch-and-bound method is more efficient for solving CSP. As a further study, we can work on integrating storage yard operations into the model. Some vessels can have large deviations from their desired berthing positions. Modifying the objective function in such a way that it evenly distributes the deviations among the vessels may be another promising research direction.

ACKNOWLEDGMENT

We gratefully acknowledge the supports of IBM through an open collaboration research award #W1056865 granted to the third author and Turkish Scientific and Technological Research Council (TÜBİTAK) through grant # 213M441.

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